University of Mumbai

Examination 2020 under cluster 4 (Lead College: PCE, New Panvel)

Examinations Commencing from 15th June 2021 to 26th June2021

Program: **Computer Engineering**

Curriculum Scheme: Rev2019

Examination: SE SemesterIII

Course Code:CSC302 and Course Name: Discrete Structures and Graph Theory

Time: 2 hour Max. Marks: 80

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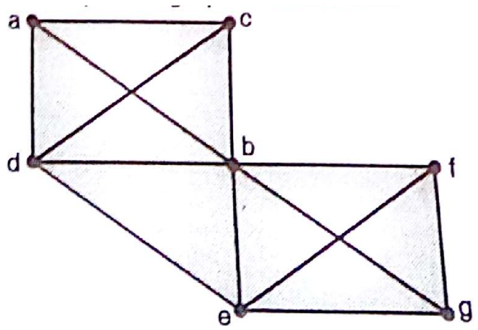
Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	The binary relation {(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2)} on the set {1, 2, 3,4} is
Option A:	Reflexiive, Symmetric and Transitive
Option B:	Irreflexive, Symmetric and Transitive
Option C:	Neither Reflexiive, nor Irreflexive but Transitive
Option D:	Irreflexive and Antisymmetric
2.	Given the following statements pick the one that is not a tautology?
Option A:	$(p\rightarrow q)\rightarrow q$
Option B:	$p\rightarrow(p\vee q)$
Option C:	$(p\wedge q)\rightarrow(p\rightarrow q)$
Option D:	$(p\wedge q)\rightarrow(p\vee q)$
3.	Given the set {1, 2, 3, 4} How many numbers must be selected from it to guarantee that at least one pair of these numbers add up to 7?
Option A:	14
Option B:	5
Option C:	9
Option D:	24
4.	All Isomorphic graph must have representation
Option A:	cyclic
Option B:	tree
Option C:	adjacency list
Option D:	adjacency matrix
5.	The cardinality of the set of odd positive integers less than 10 is ?
Option A:	5
Option B:	10
Option C:	3
Option D:	20
6.	If g(x)= 3x+2 then gog(x) :
Option A:	6x+4
Option B:	9x+8
Option C:	3x-2

Option D:	2-3x
7.	Length of path is
Option A:	Number of Edges in the path
Option B:	Number of circuits in the path
Option C:	Number of loops in the path
Option D:	Number of Vertices in the path
8.	If every two elements of a poset are comparable then the poset is called
Option A:	Sub ordered poset
Option B:	Totally ordered poset
Option C:	Sub Lattice
Option D:	Semigroup
9.	A___________ has a greatest element and a least element which satisfy 0<=a<=1 for every a in the lattice(say, L).
Option A:	semilattice
Option B:	Join semilattice
Option C:	Meet semilattice
Option D:	Bounded semilattice
10.	Let S = {a, b, c, d, e, f , g}. Determine which of the following are partitions of S:
Option A:	P1 = [{a, c, e}, {b}, {d, g}],
Option B:	P2 = [{a, e, g}, {c, d}, {b, e, f }],
Option C:	P3 = [{a, b, e, g}, {c}, {d, f }],
Option D:	P4 = [{a, b, c, d, e, f , g},{c,g}]
11.	Solution of linear homogenous recurrence relation: $a_n = 3a_{n-1} - 2a_{n-2}$ *with* $a_0 = 1,\ a_1 = 3,\ n\geq 2$ is
Option A:	$a_n = (-1) + 2^n$
Option B:	$a_n = (-1) + 3.2^n$
Option C:	$a_n = (-1)(-1)^n + 2^n$
Option D:	$a_n = (-1) + 2.2^n$
12.	The number of integers between 1 and 1000 that are divisible by 3 but not by2 or 5 is
Option A:	132
Option B:	127
Option C:	134
Option D:	143
13.	If six numbers are selected from 1 to 15,find the least number of selections which will have the same sum
Option A:	61
Option B:	91
Option C:	41
Option D:	51

14.	The number of relations from A = {a, b, c} to B = {1, 2}
Option A:	54
Option B:	74
Option C:	64
Option D:	84
15.	Let G = (Z_6,$+_6$) **is an Abelian group then the inverse element of 4 is _____.**
Option A:	0
Option B:	1
Option C:	2
Option D:	3
16.	If $G = (Z_7^*, \times_7)$ is a group , the inverse of elements 2, 3 and 6 are _____
Option A:	2,3 and 6
Option B:	1,2 and 3
Option C:	4,5 and 6
Option D:	3,4 and 6
17.	The complete graph with four vertices has _________edges.
Option A:	3
Option B:	4
Option C:	5
Option D:	6
18.	Which of the following function is bijective?
Option A:	$f: R\to R\ defined\ as\ f(x) = x^2$
Option B:	$f: R\to R\ defined\ as\ f(x) = 3^x$
Option C:	$f: R\to R\ defined\ as\ f(x) = x^3 - x$
Option D:	$f: R\to R\ defined\ as\ f(x) = x^3 + 1$
19.	**Let a POSET L,≤ be a Lattice. Then for every pair of elements a,b∈L has ______.**
Option A:	a GLB.
Option B:	a LUB.
Option C:	both GLB and LUB.
Option D:	Both Maximal and Minimal
20.	**In a graph a node which is not adjacent to any other node is called _____ node.**
Option A:	Simple
Option B:	Isolated
Option C:	Initiating

Option D:	Different

Q2	**Solve any Four out of Six 5 marks each**
A	Let A be a set of integers, Let R be a Relation on AXA defined by (a,b)R(c,d) if and only if a+d = b+c. Prove that R is an Equivalence Relation.
B	Show that the sum of the cubes of three consecutive integers is divisible by 9
C	Prove that the set A=(0,1,2,3,4,5) is a finite Abelian group under Addition modulo 6
D	Find the Transitive closure of the relation R on A={1,2,3,4} where the Relation R={(1,2),(2,2),(2,4),(3,4),(4,3),(3,2),(4,1)}
E	Check whether Euler cycle and Euler Path exists in the Graph given below. 
F	Let f : A→B be a Function from A to B . Prove that f^{-1} exists if and only if f is a Bijective Function.

Q3.	**Solve any Two Questions out of Three 10 marks each**
A	Draw the Hasse Diagram of $\mathbf{D_{72}}$ and $\mathbf{D_{105}}$ and check whether they are Lattice.
B	Consider the Set A={1,2,3,4,5,6} under multiplication Modulo 7. 1) Prove that A is a Cyclicgroup 2) Find the orders and the Subgroups generated by {2,3}and {3,4}

C	A Function $R-\left\{\frac{7}{3}\right\} \rightarrow R-\{\frac{4}{3}\}$ is defined as $f(x)=\frac{(4x-5)}{(3x-7)}$ Prove that f is Bijective and find the rule for f^{-1}

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Course Code: CSC302 and Course Name: Discrete Structures and Graph Theory

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Question Number	Correct Option (Enter either 'A' or 'B' or 'C' or 'D')
Q1.	C
Q2.	A
Q3.	B
Q4	D
Q5	A
Q6	B
Q7	A
Q8.	B
Q9.	D
Q10.	C
Q11.	D
Q12.	C
Q13.	B
Q14.	C
Q15.	C
Q16.	C
Q17.	D
Q18.	D
Q19.	A
Q20.	B

Q2) a

2a) Let A be a set of integers and R be a Relation on $A \times A$ defined by.

$(a,b) R (c,d)$ if $a+d = b+c$

prove that R is an Equivalence Relation.

Soln :- a) We have $(a,b) R (a,b)$

because $a+b = b+a$

$\therefore$ R is Reflexive.

b) if $(a,b) R (c,d)$

then $a+d = b+c$.

$\therefore d+a = c+b$.

$c+b = d+a$

$\therefore (c,d) R (a,b)$

$\therefore$ R is Symmetric

c) Let $(a,b) R (c,d)$ and. $(c,d) R (e,f)$

$\therefore a+d = b+c$ and $c+f = d+e$

Adding L.H.S and R.H.S.

$a+d+c+f = b+c+d+e$.

$a+f = b+e$.

$(a,b) R (e,f)$

$\therefore$ R is Transitive.

$\therefore$ R is an Equivalence Relation.

Q2)b

Q2b) Show that the sum of the cubes of three Consecutive Integers is divisible by 9.

Soln :- Let $P(n) = n^3 + (n+1)^3 + (n+2)^3$

Step I :- For $n = 1$

$P(1) = 1^3 + (1+1)^3 + (1+2)^3 = 1+8+27 = 36$

which is divisible by 9.

$P(2) = 2^3 + (2+1)^3 + (2+2)^3 = 8+27+64 = 99$

which is divisible by 9.

Hence $P(1)$ & $P(2)$ is True.

Step II :- Assume that the result is True for $n = K$ ie $P(K)$ is true.

$\therefore K^3 + (K+1)^3 + (K+2)^3$ is divisible by 9

$\therefore K^3 + (K+1)^3 + (K+2)^3 = 9m$ say

The next term is $P(K+1)$

Now $P(K+1) = (K+1)^3 + (K+2)^3 + (K+3)^3$

$= (K+1)^3 + (K+2)^3 + (K^3 + 9K^2 + 27K + 27)$

$= [K^3 + (K+1)^3 + (K+2)^3] + 9K^2 + 27K + 27$

$= 9m + 9(K^2 + 3K + 3)$

$P(K+1) = 9(m + K^2 + 3K + 3)$

Hence $P(K+1)$ is divisible by 9

$\therefore P(K+1)$ is true

$\therefore P(n)$ is true for $n = K+1$

Step III : Hence by Mathematical Induction the result is true for all $n \in N$.

Q2)c

Q2c) Prove that the set $A = \{0,1,2,3,4,5\}$ a finite Abelian group under Addi[tion] modulo 6.

Sol^n prepare the table

$\oplus$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

(1) From the table we see that $\oplus$ is a[ssociative]

for eg :- $2 \oplus (3 \oplus 5) = (2 \oplus 3) \oplus 5$

$2 \oplus 2 = 5 \oplus 5$

$4 = 4$

(2) The first row or the first column show[s] '0' is the identity Element

(3) The positions of '0' the additive inver[se] every row (and every column) show every element of A has the additive [inverse]. Hence inverse of 1 is 5 is

eg :- $1 \oplus 5 = 0$ $\therefore 5^{-1} = 3$

Also $3 \oplus 3 = 0$ $\therefore 3^{-1} = 3$

$2 \oplus 4 = 0$ $\therefore 2^{-1} = 4$ etc

$\therefore$ G is a group under addition mo[dulo]

(4) Further $a \oplus b = b \oplus a$

eg $4 \oplus 5 = 3$ and $5 \oplus 4 = 3$

$\therefore 4 \oplus 5 = 5 \oplus 4$. $\therefore$ G is Abelian [group]

Q2)d

2d) Find the Transitive Closure of R on $A = \{1,2,3,4\}$ where the s[et]

$R = \{(1,2), (2,2), (2,4), (3,4), (4, \ldots$

$(3,2), (4,1)\}$.

Step 1 :- $M_R = W_0 =$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 0 & 1 & 0 \end{array}$$

$C_1 = 4$ put 1's in positio[ns]

$R_1 = 2$ (4,2) (4,4)

$$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$C_2 = 1, 2, 3,$

$R_2 = 2, 4.$

put 1s in positions (1,2), (1,4), (2,2) (3,2), (3,4), (4, …

$$W_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$C_3 = 4$

$R_3 = 2, 4.$

insert 1's in pos[itions] (4,2), (4,4)

$W_3 = W_2$.

$$W_3 = W_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$C_4 = 1, 2, 3, 4.$

$R_4 = 1, 2, 3, 4.$

put 1's in all po[sitions]

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore R^{\infty} = \{(1,1), (1,2), (1,3), (1, \ldots$

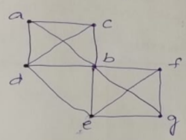
$(2,1), (2,2), (2,3), \ldots$

$(3,1), (3,2), (3,3), \ldots$

$(4,1), (4,4), (4,2) \ldots$

Q2)e

2.e) Check whether Euler cycle and Euler path exist in the graph given below.



Vertex	a	b	c	d	e	f	g
Degree	3	6	3	4	4	3	3

No of vertices with odd degree = 4.

If a graph G has a vertex of odd degree there can be no Euler cycle (circuit) in G.

If a graph G has more than two vertices of odd degree then there can be no Euler path in G

Hence the given graph has no Euler path and no Euler cycle or ckt.

Q2)f

2.f) Let $f: A \to B$ be a function from A to B. Prove that f^{-1} exists if and only if f is a Bijective function.

Proof:- f is Bijective means Every element of A is associated with some element of B and any element of A is associated with a unique element of B.

In other words f is Bijective means there is one to one correspondence between the elements of A and the elements of B.

Let a_1, a_2 be two elements of A

Let b_1, b_2 be two elements of B

such that

$f(a_1) = b_1$ and $f(a_2) = b_2$

Hence $a_1 = f^{-1}(b_1)$ and $a_2 = f^{-1}(b_2)$

if possible let $f^{-1}(b_1) = f^{-1}(b_2)$

$\therefore a_1 = a_2$

$\therefore f(a_1) = f(a_2)$

$\therefore b_1 = b_2$

$\therefore$ f^{-1} is one to one.

This means f^{-1} is one to one.

Q3b

Q3 b) Consider the set $A = \{1,2,3,4,5,6\}$ under multiplication modulo 7

i) Prove that it is a cyclic group

ii) Find the order and the subgroup generated by $\{2,3\}$ and $\{3,4\}$.

*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

1) From the first row and the first column we see that 1 is the identity element.

2) We observe that

$3^1 = 3$, $3^2 = 9_7 = 2$

$3^3 = 27_7 = 6$, $3^4 = 81_7 = 4$

$3^5 = 243_7 = 5$, $3^6 = 729_7 = 1$

Thus each element of A can be written as 3^k.

Hence $(A, *)$ is a cyclic group and 3 is its generator.

3) The subgroup generated by $\{3,4\}$ is denoted by $<\{3,4\}>$

The inverse of 3 is 5 and inverse of 4 is 2

and they belong to the subgroup $<\{3,4\}>$

The identity element 1 belongs to the subgroup

Thus the elements 1,2,3,4,5 belongs to the subgroup $<\{3,4\}>$

Now let us check whether the remaining element 6 also belongs to the subgroup.

since $4 \in <\{3,4\}>$ and $5 \in <\{3,4\}>$

$4 * 5$ must belong to the subgroup

But $(4*5)_7 = 6$ Hence $6 \in <\{3,4\}>$.

Q3)c

Example 10 : A function $R - \left\{\frac{7}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ is defined by $f(x) = \frac{4x-5}{3x-7}$.

Prove that f is bijective and find the rule for f^{-1}.

Sol. : (i) To prove that f is injective or one-to-one.

Let x_1, x_2 be two elements in $R - \left\{\frac{7}{3}\right\}$ and let $f(x_1) = f(x_2)$.

$\therefore \frac{4x_1-5}{3x_1-7} = \frac{4x_2-5}{3x_2-7}$ $\quad \therefore (4x_1-5)(3x_2-7) = (4x_2-5)(3x_1-7)$

$\therefore 12x_1x_2 - 28x_1 - 15x_2 + 35 = 12x_2x_1 - 28x_2 - 15x_1 + 35$

$\therefore (-28+15)x_1 = (-28+15)x_2$

$\therefore -13x_1 = -13x_2$ $\quad \therefore x_1 = x_2$

$\therefore$ f is injective or one-to-one.

(ii) To prove that f is surjective or onto.

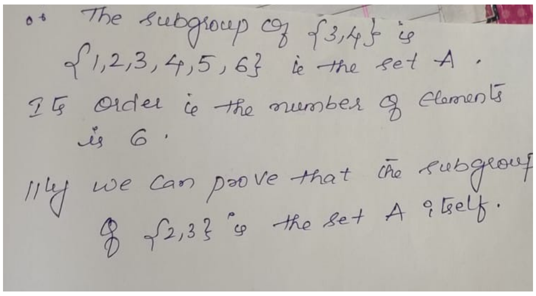
Let $y = \frac{4x-5}{3x-7}$. $\quad \therefore 3xy - 7y = 4x - 5$

$\therefore 3xy - 4x = 7x - 5$ $\quad \therefore x(3y-4) = 7y - 5$

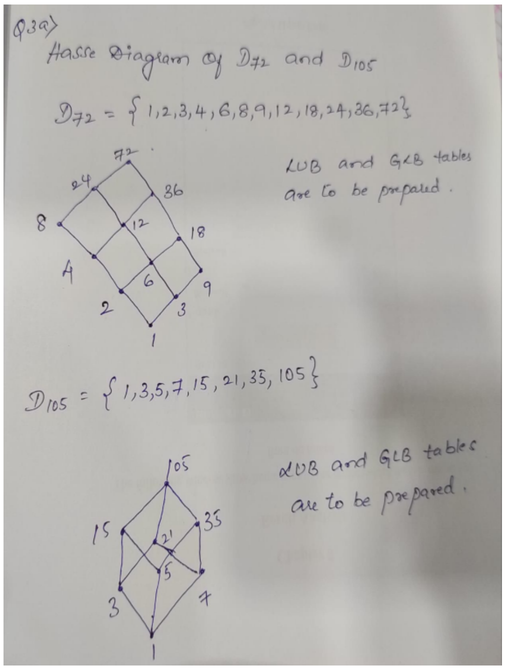
$\therefore x = \frac{7y-5}{3y-4}$ $\quad \therefore x \in R - \left\{\frac{7}{3}\right\}$ if $y \in R - \left\{\frac{4}{3}\right\}$

$\therefore$ f is surjective or onto.

(iii) Since f is injective and surjective, it is bijective and has f^{-1} and $f^{-1} = \frac{7x-5}{3x-4}$.


o. The subgroup of {3,4} is
{1,2,3,4,5,6} ie the set A.
Its order ie the number of elements
is 6.
||ly we can prove that the subgroup
of {2,3} is the set A itself.

Q3)a


Q.3a)
Hasse Diagram of D_{72} and D_{105}

D_{72} = {1,2,3,4,6,8,9,12,18,24,36,72}

LUB and GLB tables are to be prepared.

D_{105} = {1,3,5,7,15,21,35,105}

LUB and GLB tables are to be prepared.