

University of Mumbai

Examination 2021 under cluster __ (Lead College: _____)

Examinations Commencing from 1st June 2021 to 10th June 2021

Program: BE Electronics Engineering

Curriculum Scheme: Rev 2019 'C' Scheme

Examination: SE Semester IV

Course Code: ELC401 and Course Name: Engineering Mathematics IV

Time: 2 hour

Max. Marks: 80

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Note : Q1 carrying 40 marks. Q2 and Q3 are carrying 20 equal marks.

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks. 2 marks each								
1.	<p>If x is a discrete random variable with the following probability distribution</p> <table border="1" style="margin-left: auto; margin-right: auto;"><tr><td style="text-align: center;">x</td><td style="text-align: center;">1</td><td style="text-align: center;">2</td><td style="text-align: center;">3</td></tr><tr><td style="text-align: center;">$P(x)$</td><td style="text-align: center;">a</td><td style="text-align: center;">$2a$</td><td style="text-align: center;">a</td></tr></table> <p>Find $P(X \leq 2)$.</p>	x	1	2	3	$P(x)$	a	$2a$	a
x	1	2	3						
$P(x)$	a	$2a$	a						
Option A:	$\frac{1}{4}$								
Option B:	$\frac{1}{2}$								
Option C:	$\frac{3}{4}$								
Option D:	1								
2.	<p>Find $E(X)$ if X has the p.d.f $f(x) = \begin{cases} \frac{3}{4}(2x - x^2) , 0 \leq x \leq 2 \\ 0 , otherwise \end{cases}$</p>								
Option A:	$\frac{3}{2}$								
Option B:	1								
Option C:	2								
Option D:	$\frac{1}{2}$								
3.	<p>If X and Y are independent random variables with means 2,3 and variance 1,2 respectively, find the mean and variance of the random variable $Z = 2X - 5Y$</p>								
Option A:	-11 , 54								
Option B:	19 , 54								
Option C:	19 , -8								
Option D:	-11, -8								
4.	<p>Suppose the number of accidents occurring weekly on a particular stretch of a highway follow a Poisson distribution with mean 3 .Calculate the probability that there is at least one accident this week.</p>								
Option A:	0.6 347								
Option B:	0.9502								

Option C:	0.7275									
Option D:	0.8002									
5.	<p>The following results were obtained from records of age (x) and systolic blood pressure (y) of a group of 10 men:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>x</td> <td>y</td> </tr> <tr> <td>mean</td> <td>53</td> <td>142</td> </tr> <tr> <td>variance</td> <td>130</td> <td>165</td> </tr> </table> <p>Correlation coefficient = 0.8 Estimate the blood pressure of a man whose age is 45?</p>		x	y	mean	53	142	variance	130	165
	x	y								
mean	53	142								
variance	130	165								
Option A:	134.78									
Option B:	130.56									
Option C:	129.56									
Option D:	137.56									
6.	<p>A coefficient of correlation is computed to be -0.95 means that</p>									
Option A:	The relationship between the two variables is weak									
Option B:	The relationship between the two variables is strong and positive.									
Option C:	The relationship between the two variables is strong but negative.									
Option D:	The correlation coefficient cannot have this value.									
7.	<p>If the tangent of the angle made by the line of regression of y on x is 0.6 and $\sigma_x = \frac{1}{2}\sigma_y$ Find the correlation coefficient between x and y.</p>									
Option A:	- 2.5									
Option B:	0.25									
Option C:	- 0.3									
Option D:	0.3									
8.	<p>Evaluate $\int_c \frac{7z-1}{(z-3)(z+5)} dz$, where c is the circle $z = 1$.</p>									
Option A:	$2\pi i$									
Option B:	0									
Option C:	$6\pi i$									
Option D:	πi									
9.	<p>Find the residue of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ at $z = -2$</p>									
Option A:	1/9									
Option B:	5/9									
Option C:	1/3									
Option D:	4/9									
10.	<p>Identify the type of singularity of the function $f(z) = \frac{\sinh z}{z^7}$</p>									
Option A:	$z = 0$ is a pole of order 7 for the given function									
Option B:	$z = 0$ is a pole of order 6 for the given function									
Option C:	$z = 0$ is an essential singularity									

Option D:	$z = 0$ is a pole of order 3 for the given function
11.	Evaluate $\int_C \frac{e^z}{z-1} dz$ where C where c is the circle $ z = 2$.
Option A:	$2\pi i$
Option B:	$2\pi e^2$
Option C:	$2\pi e$
Option D:	πe^2
12.	Find the value of the integral $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x$
Option A:	$\frac{5-i}{6}$
Option B:	$\frac{5+i}{6}$
Option C:	$\frac{1+5i}{6}$
Option D:	$\frac{1-5i}{6}$
13.	Find the vector orthogonal to $(2, 1, -2)$ and $(1, 2, 2)$
Option A:	$(1, -2, 1)$
Option B:	$(2, -2, 1)$
Option C:	$(1, -1, 1)$
Option D:	$(2, 2, -1)$
14.	If $u = (3, 1, 4, -2)$ $v = (2, 2, 0, 1)$ then find $\langle u, v \rangle$ and $\ u\ , \ v\ $
Option A:	$-6, \sqrt{30}, \sqrt{10}$
Option B:	$5, \sqrt{2}, \sqrt{6}$
Option C:	$5, \sqrt{30}, 3$
Option D:	$6, \sqrt{30}, 3$
15	Determine which of the following are subspaces of R^3 $W_1 = \{(a, 0, b), a, b \in R\}$ $W_2 = \{(a, b, 1), a, b \in R\}$
Option A:	W_1 and W_2 are the subspaces of R^3
Option B:	W_1 and W_2 are not the subspaces of R^3
Option C:	W_1 is a subspace of R^3 but W_2 is not a subspace of R^3
Option D:	W_1 is not a subspace of R^3 but W_2 is a subspace of R^3
16.	Write down the matrix of the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_2x_3 + 8x_3x_1$
Option A:	$\begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 3 \\ 4 & 3 & -7 \end{bmatrix}$
Option B:	$\begin{bmatrix} 1 & -4 & 8 \\ -4 & 2 & 6 \\ 8 & 6 & -7 \end{bmatrix}$

Option C:	$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 2 & 3 \\ 4 & 3 & -7 \end{bmatrix}$
Option D:	$\begin{bmatrix} 1 & 4 & 8 \\ 4 & 2 & 6 \\ 8 & 6 & 7 \end{bmatrix}$
17.	Find the rank , signature, index of the transformed quadratic form $3y_1^2 + \frac{2}{3}y_2^2 - \frac{39}{2}y_3^2$.
Option A:	rank = 3, signature =2, index =1
Option B:	rank = 3, signature =1, index =2.
Option C:	rank = 2, signature =3, index =1.
Option D:	rank = 2, signatur e=1, index =3.
18.	A necessary condition for $I = \int_{x_1}^{x_2} f(x, y, y', y'')dx$ to be an extremal is that
Option A:	$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) + \frac{d^2}{dx^2} \left(\frac{\partial f}{\partial y''} \right) = 0$
Option B:	$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$
Option C:	$\frac{\partial f}{\partial y} + \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$
Option D:	$\frac{\partial f}{\partial y} + \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) + \frac{d^2}{dx^2} \left(\frac{\partial f}{\partial y''} \right) = 0$
19.	The functional $I = \int_a^b (y'^2 + 12xy)dx$ has the following extremal with c_1 and c_2 as arbitrary constants.
Option A:	$c_1x^3 + c_2x$
Option B:	$x^2 + c_1x + c_2$
Option C:	$c_1x + c_2$
Option D:	$x^3 + c_1x + c_2$
20.	The extremal of the functional $I = \int_a^b (16y'^2 - y''^2 + x^2)dx$ is
Option A:	$y = c_1 \cos 2x + c_2 \sin 2x$
Option B:	$y = c_1 e^{2x} + c_2 e^{-2x}$
Option C:	$y = c_1 e^{2x} + c_2 e^{-2x} + c_3 \cos 2x + c_4 \sin 2x$
Option D:	$y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$

Q2. (20 Marks)	Solve any Four out of Six.	5 marks each																						
A	Fit a Poisson distribution for the following distribution . <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>Total</td> </tr> <tr> <td>f</td> <td>43</td> <td>40</td> <td>25</td> <td>10</td> <td>2</td> <td>120</td> </tr> </table>	x	0	1	2	3	4	Total	f	43	40	25	10	2	120									
x	0	1	2	3	4	Total																		
f	43	40	25	10	2	120																		
B	Obtain the rank correlation coefficient for the following data <table border="1" style="margin-left: 20px;"> <tr> <td>X</td> <td>68</td> <td>64</td> <td>75</td> <td>50</td> <td>64</td> <td>80</td> <td>75</td> <td>40</td> <td>55</td> <td>64</td> </tr> <tr> <td>Y</td> <td>62</td> <td>58</td> <td>68</td> <td>45</td> <td>81</td> <td>60</td> <td>68</td> <td>48</td> <td>50</td> <td>70</td> </tr> </table>	X	68	64	75	50	64	80	75	40	55	64	Y	62	58	68	45	81	60	68	48	50	70	
X	68	64	75	50	64	80	75	40	55	64														
Y	62	58	68	45	81	60	68	48	50	70														
C	Obtain two distinct Laurent's series of $f(z) = \frac{2z-3}{z^2-4z+3}$ about $z = 4$ indicating the region of convergence																							
D	Construct an orthonormal basis of R^3 using Gram-Schmidt process to $S = \{(1,0,0), (3, 7, -2), (0,4,1)\}$																							
E	Reduce the symmetric matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to the diagonal form using congruent transformation and interpret the result in terms of quadratic forms																							
F	Find the curve on which the functional $\int_a^b \sqrt{1+y'^2} dx$ is extremum.																							

Q3. (20 Marks)	Solve any Four out of Six.	5 marks each
A	In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5 Assuming the distribution to be normal ,find (i)how many students score between 12 and 15 ? (ii) how many score above 18? (iii) how many score below 8?	
B	In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible: $\sigma_x = 3$. Regression equations: $8X-10Y = -66$, $40X-18Y=214$. What are: (i) the mean values X and Y, (ii) the correlation coefficient between X and Y, (iii) the standard deviation of Y	
C	Evaluate $\oint_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-2)(z-3)} dz$ where C is the circle $ z =4$.	
D	Let V be a set of positive real numbers with addition and scalar multiplication defined as $x + y = xy$ and $cx = x^c$.Show that V is a vector space under this addition and scalar multiplication.	
E	Reduce the following quadratic form into canonical form. $Q: x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_3 + 2x_2x_3 + 2x_2x_1$	
F	Using Rayleigh -Ritz method , solve the boundary value problem $I = \int_0^1 (y'^2 - y^2 - 2xy) dx$ with $y(0)=0$ and $y(1)=0$.	

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Time: 2 hour

Max. Marks: 80

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Question Number	Correct Option (Enter either 'A' or 'B' or 'C' or 'D')
Q1.	C
Q2.	B
Q3.	A
Q4	B
Q5	A
Q6	C
Q7	D
Q8.	B
Q9.	D
Q10.	B
Q11.	C
Q12.	A
Q13.	B
Q14.	D
Q15.	C
Q16.	A
Q17.	B
Q18.	A
Q19.	D
Q20.	C