

**K. J. Somaiya Institute of Engineering and Information Technology, Sion, Mumbai-22**  
**(Autonomous College Affiliated to University of Mumbai)**

**End Semester Exam**  
 Nov -Dec (2021-2022)

**(B.Tech.) Program: Computer Engineering**

Examination: **TY** Semester: **V**

Course Code: **1UCEDLC5054** and Course Name: **Probabilistic Graphical Models**

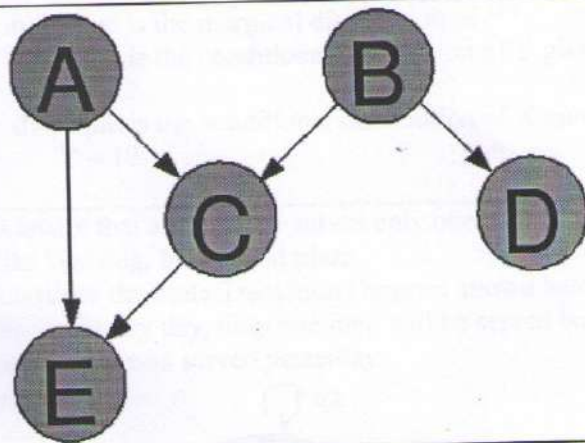
Duration: 03 Hours

Max. Marks: 60

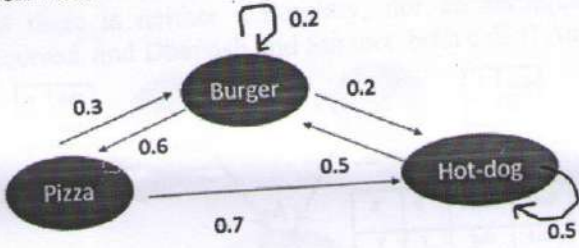
Instructions:

- (1) All questions are compulsory.
- (2) Draw neat diagrams wherever applicable.
- (3) Assume suitable data, if necessary
- (4) Use of Calculator is allowed.

|            |   | Max. Marks    | CO            | BT level      |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
|------------|---|---------------|---------------|---------------|---|---|-----|---|---|-----|---|---|-----|---|---|-----|---|---|---------------|---|---|-----|---|---|-----|---|---|-----|---|---|-----|---|---|---|---------------|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|--|---|---|---|--|---|---|---|--|---|---|---|--|---|---|---|---|----|-----|----|
| <b>Q 1</b> | <b>Solve any six questions out of eight:</b>  | <b>12</b>     |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| i)         | Explain Graphs and its related terminologies, Explain Cliques with example.   | 2M            | CO1           | U             |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| ii)        | Explain the concept of I-Equivalence in Bayesian Networks with examples.  | 2M            | CO2           | U             |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| iii)       | Let X, Y and Z be binary variables. Compute the selected entries (marked by a '?') in the factor $\psi(X,Y,Z)=\phi_1(X,Y)\cdot\phi_2(Y,Z)$ ,<br><br><div style="display: flex; align-items: center; justify-content: center;"> <table border="1" style="margin-right: 10px;"> <thead> <tr><th>X</th><th>Y</th><th><math>\phi_1(X,Y)</math></th></tr> </thead> <tbody> <tr><td>1</td><td>1</td><td>0.8</td></tr> <tr><td>1</td><td>2</td><td>0.5</td></tr> <tr><td>2</td><td>1</td><td>0.5</td></tr> <tr><td>2</td><td>2</td><td>0.6</td></tr> </tbody> </table> <span style="margin: 0 10px;">×</span> <table border="1" style="margin-right: 10px;"> <thead> <tr><th>Y</th><th>Z</th><th><math>\phi_2(Y,Z)</math></th></tr> </thead> <tbody> <tr><td>1</td><td>1</td><td>0.2</td></tr> <tr><td>1</td><td>2</td><td>0.2</td></tr> <tr><td>2</td><td>1</td><td>0.9</td></tr> <tr><td>2</td><td>2</td><td>1.0</td></tr> </tbody> </table> <span style="margin: 0 10px;">=</span> <table border="1" style="margin-right: 10px;"> <thead> <tr><th>X</th><th>Y</th><th>Z</th><th><math>\psi(X,Y,Z)</math></th></tr> </thead> <tbody> <tr><td>1</td><td>1</td><td>1</td><td>?</td></tr> <tr><td>1</td><td>1</td><td>2</td><td></td></tr> <tr><td>1</td><td>2</td><td>1</td><td>?</td></tr> <tr><td>1</td><td>2</td><td>2</td><td></td></tr> <tr><td>2</td><td>1</td><td>1</td><td></td></tr> <tr><td>2</td><td>1</td><td>2</td><td></td></tr> <tr><td>2</td><td>2</td><td>1</td><td></td></tr> <tr><td>2</td><td>2</td><td>2</td><td>?</td></tr> </tbody> </table> </div> | X             | Y             | $\phi_1(X,Y)$ | 1 | 1 | 0.8 | 1 | 2 | 0.5 | 2 | 1 | 0.5 | 2 | 2 | 0.6 | Y | Z | $\phi_2(Y,Z)$ | 1 | 1 | 0.2 | 1 | 2 | 0.2 | 2 | 1 | 0.9 | 2 | 2 | 1.0 | X | Y | Z | $\psi(X,Y,Z)$ | 1 | 1 | 1 | ? | 1 | 1 | 2 |  | 1 | 2 | 1 | ? | 1 | 2 | 2 |  | 2 | 1 | 1 |  | 2 | 1 | 2 |  | 2 | 2 | 1 |  | 2 | 2 | 2 | ? | 2M | CO3 | Ap |
| X          | Y   | $\phi_1(X,Y)$ |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 1   | 0.8           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 2   | 0.5           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 1   | 0.5           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 2   | 0.6           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| Y          | Z   | $\phi_2(Y,Z)$ |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 1   | 0.2           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 2   | 0.2           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 1   | 0.9           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 2   | 1.0           |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| X          | Y   | Z             | $\psi(X,Y,Z)$ |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 1   | 1             | ?             |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 1   | 2             |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 2   | 1             | ?             |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 1          | 2   | 2             |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 1   | 1             |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 1   | 2             |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 2   | 1             |               |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| 2          | 2   | 2             | ?             |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| iv)        | What is the Temporal Markov Model? Explain with a diagram   | 2M            | CO4           | U             |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| v)         | Explain the terms Expected Utility and Expected Value with example.   | 2M            | CO5           | U             |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| vi)        | List any four applications of Markov Networks.  | 2M            | CO6           | U             |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |
| vii)       | Assume that the variable E is observed. A, B, C, and D are not observed. Which pairs of variables other than E are independent in the given graphical model, given E? Justify your answer.  | 2M            | CO2           | Ap            |   |   |     |   |   |     |   |   |     |   |   |     |   |   |               |   |   |     |   |   |     |   |   |     |   |   |     |   |   |   |               |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |  |   |   |   |   |    |     |    |

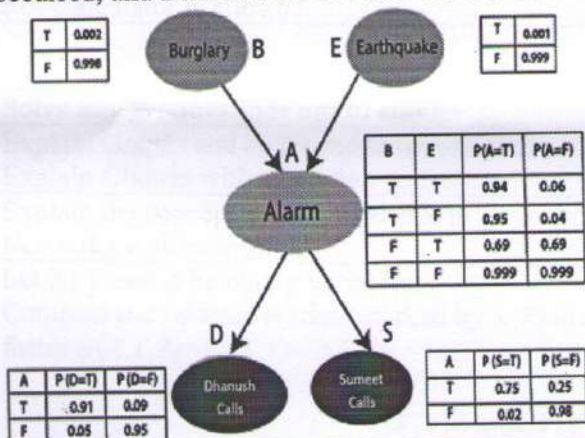


| viii)      | Explain the importance of Reduced Markov Networks.   | 2M        | CO3  | U    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
|------------|--|-----------|------|------|--|--|---|---|---|---|---|-------|---|---|---|-----|------|---|---|-----|-----|------|---|-----|------|------|----|-----|----|
| <b>Q.2</b> | <b>Solve any four questions out of six.</b>  | <b>16</b> |      |      |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| i)         | Explain Subjective and Objective probability concepts with suitable examples.  | 4M        | CO1  | U    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| ii)        | <p>In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables A (alarm sounds), FA (alarm is faulty), and FG (gauge is faulty) and the multivalued, discrete nodes G (gauge reading) and T (actual core temperature).</p> <p>Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.</p>  | 4M        | CO2  | Ap   |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| iii)       | What is log linear model parameterization? Explain its mathematical derivation.  | 4M        | CO3  | U    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| iv)        | Explain the properties of Hidden Markov Model with an example.   | 4M        | CO4  | U    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| v)         | Explain the relationship between Correlation and Causation.  | 4M        | CO5  | U    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| vi)        | Explain Image Segmentation using Probabilistic Models.   | 4M        | CO6  | U    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| <b>Q.3</b> | <b>Solve any two questions out of three.</b>   | <b>16</b> |      |      |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| i)         | <p>For the joint probability distribution table given below:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2" rowspan="2"></th> <th colspan="3">X</th> </tr> <tr> <th>0</th> <th>1</th> <th>2</th> </tr> </thead> <tbody> <tr> <th rowspan="4">Y</th> <th>0</th> <td>1/120</td> <td>0</td> <td>0</td> </tr> <tr> <th>1</th> <td>1/8</td> <td>1/10</td> <td>0</td> </tr> <tr> <th>2</th> <td>1/4</td> <td>1/4</td> <td>1/24</td> </tr> <tr> <th>3</th> <td>1/8</td> <td>1/20</td> <td>1/20</td> </tr> </tbody> </table> <p>a) What is the marginal distribution of X?</p> |           |      | X    |  |  | 0 | 1 | 2 | Y | 0 | 1/120 | 0 | 0 | 1 | 1/8 | 1/10 | 0 | 2 | 1/4 | 1/4 | 1/24 | 3 | 1/8 | 1/20 | 1/20 | 8M | CO1 | Ap |
|            |  |           |      | X    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
|            |  | 0         | 1    | 2    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
| Y          | 0  | 1/120     | 0    | 0    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
|            | 1  | 1/8       | 1/10 | 0    |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
|            | 2  | 1/4       | 1/4  | 1/24 |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |
|            | 3  | 1/8       | 1/20 | 1/20 |  |  |   |   |   |   |   |       |   |   |   |     |      |   |   |     |     |      |   |     |      |      |    |     |    |

|            |  |           |     |    |
|------------|--|-----------|-----|----|
|            | <p>b) What is the marginal distribution of Y?</p> <p>c) What is the conditional distribution of Y given <math>X = 2</math>?</p> <p>d) What is the conditional distribution of X given <math>Y = 1</math>?</p>  |           |     |    |
| ii)        | <p>Assume that a restaurant serves only one item per day like Hot-dog, burger and pizza. Consider the State Transition Diagram shown below. Rule: On any day, only one item will be served based upon what was served yesterday.</p>  <p>Find the probability that the 4th item after the sequence {Pizza, Burger, Pizza} will be Hot-dog. Find the Stationary States matrix.</p>   | 8M        | CO4 | Ap |
| iii)       | <p>You have an opportunity to place a bet on the outcome of an upcoming race involving a certain female horse named Bayes: if you bet <math>x</math> dollars and Bayes wins, you will have <math>w_0 + x</math>, while if she loses you will have <math>w_0 - x</math>, where <math>w_0</math> is your initial wealth.</p> <p>1. Suppose that you believe the horse will win with probability <math>p</math> and that your utility for wealth <math>w</math> is <math>\ln(w)</math>. Find your optimal bet as a function of <math>p</math> and <math>w_0</math>.</p> <p>2. You know little about horse racing, only those racehorses are either winners or average, that winners win 90% of their races, and that average horses win only 10% of their races. After all the buzz you've been hearing, you are 90% sure that Bayes is a winner. What fraction of your wealth do you plan to bet?</p> <p>3. As you approach the betting window at the track, you happen to run into your uncle. He knows rather a lot about horse racing: he correctly identifies a horse's true quality 95% of the time. You relay your excitement about Bayes. "Don't believe the hype," he states. "That Bayes mare is only an average horse." What do you bet now (assume that the rules of the track permit you to receive money only if the horse wins)?</p> | 8M        | CO5 | Ap |
| <b>Q.4</b> | <b>Solve any two questions out of three.</b>   | <b>16</b> |     |    |
| i)         | <p>Amit installed a new burglary alarm at his home to detect burglary. The alarm reliably responds at detecting a burglary but also responds for minor</p>   | 8M        | CO2 | Ap |

earthquakes. Harry has two neighbours Dhanush and Sumeet, who have taken a responsibility to inform Amit at work when they hear the alarm. Dhanush always calls Amit when he hears the alarm, but sometimes she gets confused with the phone ringing and calls at that time too. On the other hand, Sumeet likes to listen to high music, so sometimes he misses hearing the alarm. Here we would like to compute the probability of Burglary Alarm.

Calculate the probability that the alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and Dhanush and Sumeet both called Amit.



|      |  |    |     |   |
|------|--|----|-----|---|
| ii)  | Explain Local Markov Assumptions? How can we construct Minimal I-maps for the same distribution? | 8M | CO3 | U |
| iii) | Explain application of Hidden Markov Model for Part of Speech Tagging.                           | 8M | CO6 | U |