

K. J. Somaiya Institute of Engineering and Information Technology, Sion, Mumbai-22

(Autonomous College Affiliated to University of Mumbai)

End Semester Exam

~~Nov. - Dec.~~

~~April - May~~ 2021-22

(conducted in March 2022)

Program: B.Tech

Examination: FY Semester: I

Course Code: 1UBSC101 and Course Name: Engineering Mathematics I

Duration: 03 Hours

Max. Marks: 60

Instructions:

- (1) All questions are compulsory.
- (2) Draw neat diagrams wherever applicable.
- (3) Assume suitable data, if necessary.

		Max. Marks	CO	BT level
Q 1	Solve any six questions out of eight:	12		
i)	Find the product of values of $i^{1/4}$.	02	CO1	2
ii)	If $x - \frac{1}{x} = 2i \sin \theta$, $y - \frac{1}{y} = 2i \sin \phi$, then find the value of $xy - \frac{1}{xy}$.	02	C01	2
iii)	Show that $\tanh(\log 5) = \frac{2}{3}$	02	C02	2
iv)	Separate into real and imaginary parts of $i^{\log(1+i)}$	02	C02	2
v)	If $u = x^3 y e^z$, where $x = t$, $y = t^2$, $z = \log t$ find $\frac{\partial u}{\partial t}$ at $t = 2$.	02	C03	2
vi)	Find all the stationary points of $f(x, y) = xy(3 - x - y)$.	02	C04	2

vii)	Determine whether the following matrix is orthogonal or not $A = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$ hence find A^{-1} .	02	C05	2
viii)	Find rank of the matrix using echelon form $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$	02	C05	2
Q.2	Solve any four questions out of six.	16		
i)	If $\sin 6\theta = a \cos^5 \theta \sin \theta + b \cos^3 \theta \sin^3 \theta + c \cos \theta \sin^5 \theta$, then find a, b, c.	04	C01	3
ii)	Prove that $\tanh^{-1} x = \sinh^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$	04	C02	3
iii)	If $u = \sinh^{-1} \left(\frac{x^3+y^3}{x^2+y^2} \right)$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\tanh^3 u$.	04	C03	3
iv)	If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, then prove that $\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} + \frac{1}{z} \frac{\partial u}{\partial z} = 0$	04	C03	3
v)	If $y = \frac{x^2+4}{(x-1)^2(2x+3)}$, find y_n .	04	C04	3
vi)	Find the rank of the following matrix by reducing to normal form. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$	04	C05	3
Q.3	Solve any two questions out of three.	16		

i)	Prove that $2(1 + \cos 8\theta) = (x^4 - 4x^2 + 2)^2$, where $x = 2 \cos \theta$.	08	CO1	3.
ii)	Show that $\tan^{-1} \left(\frac{x+iy}{x-iy} \right) = \frac{\pi}{4} + \frac{i}{2} \log \left(\frac{x+y}{x-y} \right)$	08	CO2	3.
iii)	If $y = \sqrt{\frac{1+x}{1-x}}$, prove that $(1 - x^2)y_n - [2(n - 1)x + 1]y_{n-1} - (n - 1)(n - 2)y_{n-2} = 0$	08	CO4	3.
Q.4	Solve any two questions out of three.	16		
i)	If $u = \frac{x^4+y^4}{x^2y^2} + x^6 \tan^{-1} \left(\frac{x^2+y^2}{x^2+2xy} \right)$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ at $(x, y) = (1, 2)$	08	CO3	3.
ii)	Find the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$.	08	CO4	3
iii)	Show that for $\mu = 0$, the system of equations $2x + y = a$, $x + \mu y - z = b$, $y + 2z = c$ has unique solution for every choice of a, b and c . If $\mu = 0$, determine the relation satisfied by a, b, c such that the system is inconsistent.	08	CO5	3