

K. J. Somaiya Institute of Engineering and Information Technology, Sion, Mumbai-22

(Autonomous College Affiliated to University of Mumbai)

End Semester Exam

Feb 2022-23

Program: B.Tech

Examination: FY Semester: I

Course Code: BSC101 and Course Name: Engineering Mathematics I

Duration: 2 Hours 30 Minutes

Max. Marks: 60

Instructions:

- (1) All questions are compulsory.
- (2) Draw neat diagrams wherever applicable.
- (3) Assume suitable data, if necessary. All logarithms are taken with base  $e$ .

		Max. Marks	CO	BT level
Q 1	Solve any six questions out of eight:	12		
i)	Evaluate $(1 + i)^{100} + (1 - i)^{100}$ .	02	C01	A
ii)	Find all the roots of the equation $x^4 - 1 = 0$ .	02	C01	A
iii)	Find the value of $\tanh(\log x)$ , if $x = \sqrt{3}$ . Also find the value of $\tanh(\log \sqrt{5})$ .	02	C02	A
iv)	Prove that $\log(1 + i) + \log(1 - i) = \log 2$ .	02	C02	A
v)	If $x = r \cos \theta$ and $y = r \sin \theta$ prove that $\left(\frac{\partial x}{\partial \theta}\right)_r = r^2 \left(\frac{\partial \theta}{\partial x}\right)_y$ .	02	C03	A
vi)	If $u = x^3 \sin^{-1}\left(\frac{\sqrt{y}}{\sqrt{x}}\right)$ , find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ .	02	C03	A

vii)	Find $n^{\text{th}}$ derivative of $y = \sin x \cos 3x$ .	02	C04	A
viii)	Determine whether the following matrix is orthogonal? $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$	02	C05	A
Q.2	Solve any four questions out of six.	16		
i)	Show that the continued product of all the values of $i^{2/3}$ is -1.	04	C01	A
ii)	If $\log(\log(x + iy)) = p + iq$ , then prove that $y = x \tan(\tan q \cdot \log \sqrt{x^2 + y^2})$	04	C02	A
iii)	If $u = \frac{1}{r}$ and $r^2 = x^2 + y^2 + z^2$ , prove that $xu_x + yu_y + zu_z + u = 0.$	04	C03	A
iv)	If $y = \tan^{-1} \left[ \frac{a+x}{a-x} \right]$ , prove that $(a^2 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$	04	C04	A
v)	Find the points on the curve $xy^2 = 54$ which are nearest to the origin.	04	C04	A
vi)	Reduce the following matrix to Row-Echelon form and find their rank. $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$	04	C05	A
Q.3	Solve any two questions out of three.	16		
i)	If $\alpha, \beta$ are the roots of $z^2 \sin^2 \theta - z \cdot \sin 2\theta + 1 = 0$ , then prove that $\alpha^n + \beta^n = 2 \cos n\theta \operatorname{cosec} n\theta$	08	CO1	A

ii)	Separate into real and imaginary parts of $\tan^{-1}(e^{i\theta})$ .	08	CO2	A
iii)	Investigate for what value of $\lambda$ , the equations below has a solution and solve them in each case. $x + 2y + z = 3$ , $x + y + z = \lambda$ , $3x + y + 3z = \lambda^2$ .	08	CO5	A
Q.4	Solve any two questions out of three.	16		
i)	Verify Euler's theorem for Homogeneous functions for function $u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$ .	08	CO3	A
ii)	If $y = x \log \left( \frac{x-1}{x+1} \right)$ , prove that $y_n = (-1)^{n-2} (n-2)! \left[ \frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n} \right]$ .	08	CO4	A
iii)	Find non-singular matrices $P$ and $Q$ such that $PAQ$ is in normal form. Also find the rank of $A$ and find $A^{-1}$ .	08	CO5	A