K. J. Somaiya Institute of Technology, Sion, Mumbai-22
(Autonomous College Affiliated to University of Mumbai)
April – May 2023
Program: B.Tech Scheme : II
Examination: SY Semester: IV Course Code: CEC401, AIC401, ITC401
Course Name: Applications of Mathematics in Engineering-II

Date of Exam: 13/05/2023 **Duration: 2.5 Hours** **Max. Marks: 60**

Instructions:
(1)All questions are compulsory.
(2)Draw neat diagrams wherever applicable.
(3)Assume suitable data, if necessary.
(4) Use of Statistical Tables permitted.

		Max. Marks	CO	BT level
Q 1	Solve any six questions out of eight:	12		
i)	Find the eigenvalues of adj. A if $A=\begin{bmatrix}-1 & 1 & -2\\ 0 & -3 & 4\\ 0 & 0 & -2\end{bmatrix}$	2	1	Ap
ii)	Evaluate $\int f(z)dz$ along the parabola $y=2x^2$ from $z=0$ to $z=3+18i$ where $f(z)=x^2-2iy$	2	2	Ap
iii)	Find the angle between the vectors (2,-1,1) and (1,1,2)	2	3	Ap
iv)	A variable X follows a Poisson distribution with variance 3. Calculate (i) P(X=2) (ii) P(X ≥ 4)	2	4	Ap
v)	The incomes of a group of 10,000 persons were found to be normally distributed with mean ₹ 520 and standard deviation ₹ 60 . Find the number of persons having incomes between ₹400 and ₹550	2	4	Ap
vi)	Write the dual of following LPP Maximise $z=2x_1-x_2+4x_3$ Subject to $x_1+2x_2-x_3\le 5, 2x_1-x_2+x_3\le 6, x_1+x_2+3x_3\le 10$ $4x_1+x_3\le 12, x_1,x_2,x_3\ge 0$	2	5	Ap
vii)	Find the stationary point of the function $z=x_1{}^2+x_2{}^2+x_3{}^2-4x_1-8x_2-12x_3+100$	2	6	Ap
viii)	If $A=\begin{bmatrix}123 & 231 & 312\\ 231 & 312 & 123\\ 312 & 123 & 231\end{bmatrix}$, prove that one of the eigenvalues of A is 666.	2	1	Ap
Q.2	Solve any four questions out of six.	16		
i)	Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ and obtain A^{-1}	4	1	Ap

ii)	Using Cauchy's residue theorem evaluate $\int_C \frac{\sin^6 z}{(z-\frac{\pi}{6})^n} dz$ where C is the circle $\|z\| = 1$ for n=1 and n=3	4	2	Ap
iii)	Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1,u_2,u_3\}$ into orthonormal basis, where $u_1=(0,0,1), u_2=(0,1,1), u_3=(1,1,1)$.	4	3	Ap
iv)	*Maximise* $z = x_1 + 3x_2 + 3x_3$ Subject to $x_1 + 2x_2 + 3x_3 = 4$ $2x_1 + 3x_2 + 5x_3 = 7$. Find all the basic solutions to the above problem. Which of them is basic feasible, non- degenerate, infeasible basic and optimal solution?	4	5	Ap
v)	Using Lagrange's multipliers, solve the following NLPP Optimise $z = 7{x_1}^2 + 5{x_2}^2$ Subject to $2x_1 + 5x_2 = 7$, $x_1, x_2 \geq 0$.	4	6	Ap
vi)	Samples of electric tubes of two companies were tested for lengths of their life and the following information was obtained , <table><tr><td></td><td>Company A</td><td>Company B</td></tr><tr><td>No. of sample</td><td>8</td><td>7</td></tr><tr><td>Mean life in hrs</td><td>1134</td><td>1024</td></tr><tr><td>Standard deviations in hrs</td><td>35</td><td>40</td></tr></table>Find the t statistic to test whether the difference in the sample means is significant	4	4	Ap
Q.3	Solve any two questions out of three.	16		
i)	Show that the following matrix is diagonalisable. Also find the diagonal form and a diagonalising matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$	8	1	Ap
ii)	If $f(\xi) = \int_C \frac{3z^2+7z+1}{z-\xi} dz$ where C is the circle $\|z\| = 2$, find the values of $f(-3), f(i), f'(1-i), f''(1-i)$	8	2	Ap
iii)	Using Big M method solve the following LPP Maximize $z = 5x_1 - 2x_2 + 7x_3$ Subject to $2x_1 + 2x_2 - x_3 \geq 2$, $3x_1 - 4x_2 \leq 3$; $x_2 + 3x_3 \leq 5$; $x_1, x_2, x_3 \geq 0$	8	5	Ap
Q.4	Solve any two questions out of three.	16		
i)	If $V = R^3$ is the vector space of ordered triplets of real numbers with usual addition and scalar multiplication .Show whether the following are subspaces of V or not (a) $W = \{(p,q,r): q = p + r, p,q,r \in R\}$	8	3	Ap

<table>
<tr><td></td><td>(b) $W = \{(x, y, z): y = x + z, x, y, z \in R\}$</td><td></td><td></td><td></td></tr>
<tr><td>ii)</td><td>300 digits were chosen at random from a table of random numbers. The frequencies of digits were as follows.
<table>
<tr><td>Digit:</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>Total</td></tr>
<tr><td>Frequency:</td><td>28</td><td>29</td><td>33</td><td>31</td><td>26</td><td>35</td><td>32</td><td>30</td><td>31</td><td>25</td><td>300</td></tr>
</table>
Use χ^2-test examine the hypothesis that the digits were distributed in equal numbers in the table.</td><td>8</td><td>4</td><td>Ap</td></tr>
<tr><td>iii)</td><td>Using the Kuhn –Tucker conditions to solve the N.L.P.P
Maximise $z = 8x_1 + 10x_2 - x_1^2 - x_2^2$
Subject to $3x_1 + 2x_2 \le 6$; $x_1, x_2 \ge 0$.</td><td>8</td><td>6</td><td>Ap</td></tr>
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