K. J. SOMAIYA INSTITUTE OF MANAGEMENT STUDIES AND RESEARCH, Vidyavihar, Mumbai- 400077

Program: ***MCA, Sem-I (2017-20 Batch)***
Subject: Mathematical Foundation in Computer Science
(End Term Exam)

Maximum Marks: 50
Duration: 3 hrs. **Date: 23rd November, 2017**

Instructions

1. Attempt any five questions

QUESTION 1
(3+5+2)

a. Suppose that the value of ‘P→Q’ is ‘true’. What can be said about the value of $\sim P \wedge Q \leftrightarrow P \vee Q$?
b. Using mathematical induction prove that
$$B \cup \left(\bigcap_{i=1}^{n} A_i \right) = \bigcap_{i=1}^{n} (B \cup A_i)$$
for every positive integer n.
c. Let A={a, b, c, d, e} and R={(a,a), (a,b), (b,c), (c,e), (c,d), (d,e)}. Draw digraph of R. Compute R^2.

QUESTION 2
(5+5)

a. Test the validity of the argument:
“If there was a ball game, then travelling was difficult. If they arrive on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game.”
b. Let S= {x| x is real number not equal to 1 or 0} and let G={f_1, f_2, f_3, f_4, f_5, f_6} be a set of six functions defined as below.

$f_1(x) = x$, $f_2(x) = 1 - x$, $f_3(x) = 1/x$, $f4(x) = 1/(1-x)$, $f_5(x) = 1-(1/x)$, $f_6 = x/(x-1)$.

Show that (G, *) is group under the operation of composition of functions. Obtain the composition table of (G, *).

QUESTION 3

(5+5)

a. Let $V = \{ v_0, w, a, b, c\}$, $S = \{ a, b, c\}$ and let $\mapsto$ be the relation on V^* given by

1. $v_0 \mapsto aw$ 2. $w \mapsto bbw$ 3. $w \mapsto c$.

Consider the phase structure grammar $G = (V, S, v_0, \mapsto)$.

i. Derive the sentence ab^6c. Also draw the derivation tree.

ii. Derive the sentence ab^4c. Also draw the derivation tree.

b. Find the particular solution for the recurrence relation $a_n + 2a_{n-1} + 2a_{n-2} = 2^n$

QUESTION 4

(4+3+3)

a. Consider a set A = { a, b, c, d, e, f} and a relation R defined on A given by R = { (a. a), (a, b), (b, a) (b, b), (c, c), (d, d), (d, e), (d, f), (e, d), (e, e), (e, f), (f, d), (f, e), (f, f) }. Write the matrix representation M_R of the relation and hence prove that it is an equivalence relation by matrix method

b. Draw the state transition diagram for the following

$S = \{s_0, s_1, s_2, s_3\}$, $I = \{a, b, c\}$

	a	b	c
S_0	S_0	S_0	S_0
S_1	S_2	S_3	S_2
S_2	S_1	S_0	S_3
S_3	S_3	S_2	S_3

c. Find the conjunctive normal form of $(p \vee q) \leftrightarrow (p \wedge q)$ by laws of logic.

QUESTION 5 (5+5)

a. Let $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix.

Determine the (3, 6) encoding function $e_H: B^3 \rightarrow B^6$. Decode the word 001111 relative to a maximum likelihood decoding function associated with e_H.

b. For the grammar specified below describe precisely the language, L(G), produced . Also give the BNF and the corresponding syntax diagram for the production of the production of the Grammar.

$G = (V, S, v_0, \mapsto)$

$V = \{v_0, a, b\}$, $S = \{a, b\}$

$v_0 \mapsto aav_0$

v ↦a
v0↦b
0

QUESTION 6 (5+5)

a. Let A = R (set of all real numbers). We define the following relation R on A. xRy iff x and y satisfy the equation $\frac{x^2}{4}+\frac{y^2}{9}=1$.Find R(x) and R(1).
b. Let A={1,2,3,5,6,10,15,30} and let R be the relation "divides". Draw the Hasse diagram of the poset (A,R) and if B={2,6,15} , find all upper bounds and lower bounds , LUB & GLB.

****