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**Vidyavihar, Mumbai- 400077**

**Program: MCA, Sem-I (2016-19Batch)**  
**Subject: Mathematical Foundation in Computer Science**  
**(End Term Exam)**

**Maximum Marks: 50**

**Duration: 3 hrs.**

**2016**

**Date: 21<sup>st</sup> November**

**Instructions**

**1. Attempt any five questions**

**QUESTION 1**

(10)

- a. Prove that for any positive integer 'n' the number ' $n(n^2 + 5)$ ' is an integer multiple of 6 by mathematical induction
- b. If I be the set of integers and if R be defined over I by "a R b" iff "a-b is an even integer", a, b ∈ I, then show that the relation R is an equivalence relation

**QUESTION 2**

(10)

- a. Determine whether the set of even integers with binary operation defined as  $a*b = (ab)/2$  is a semigroup, a monoid or neither. If it is a monoid, specify the identity. If it is a semigroup or a monoid determine whether it is commutative.
- b. Check the validity of the following arguments.  
If I play football, I cannot study.  
Either I study or I pass DBMS  
I played football.  
Therefore, I passed DBMS.

**QUESTION 3**

(10)

- a. Define degree of a vertex in an undirected graph. How many edges are there in a graph with ten vertices each of degree six?

- b. State the Tower of Hanoi problem. Obtain its recurrence relation with suitable initial conditions. Solve the recurrence relation.

#### **QUESTION 4**

(10)

- a. Draw the graph for following Adjacency list & write Euler Circuit and Hamilton Circuit if exists.

A	B	C	E
B	A	D	F
C	A	D	G
D	B	C	H
E	A	F	G
F	B	E	H
G	C	E	H
H	D	F	G

- b. Let the state transition table for a finite state machine be

	0	1
$S_0$	$S_0$	$S_1$
$S_1$	$S_1$	$S_2$
$S_2$	$S_3$	$S_2$
$S_3$	$S_3$	$S_4$
$S_4$	$S_4$	$S_2$

List values of the transition function  $f_w$  for (i)  $w = 01011$ , (ii)  $w = 10101$ .

#### **QUESTION 5**

(10)

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- a. Let  $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  be a parity check matrix.

Determine the  $(3, 6)$  encoding function  $e_H: B^3 \rightarrow B^6$ . Decode the word 001111 relative to a maximum likelihood decoding function associated with  $e_H$ .

- b. Find the solution of the recurrence relation  $a_n = 4a_{n-1} + 1$  with  $a_1 = 4$  as initial condition.

**QUESTION 6**

(10)

by laws of logic

$$(p \vee q) \leftrightarrow (p \wedge q).$$

a. Obtain the following

Principal conjunctive normal form of  $(p \leftrightarrow q) \vee \sim r$

(i) Conjunctive normal form of

Disjunctive normal form of  $\sim(p \vee q) \leftrightarrow (p \wedge q)$

(ii)

(iii)

b. Let  $V = \{v_0, v_1, v_2, a, b, c\}$ ,  $S = \{a, b, c\}$  and let  $\mapsto$  be the relation on  $V^*$  given by

1.  $v_0 \mapsto v_0 v_1$  2.  $v_0 v_1 \mapsto b b w v_2 v_0$  3.  $v_2 \mapsto a, v_2 v_0 \mapsto a b, v_1 \mapsto c$

Consider the phase structure grammar  $G = (V, S, v_0, \mapsto)$ .

(i) Derive the sentence xyz. Also draw the derivation tree.

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