## University of Mumbai

## Examination 2020 under cluster 4 (Lead College: PCE, New Panvel)

Examinations Commencing from 15 ${ }^{\text {th }}$ June 2021 to $\mathbf{2 6}^{\text {th }}$ June2021
Program: Computer Engineering
Curriculum Scheme: Rev 2019
Examination: SE Semester III
Course Code: CSC302 and Course Name: Discrete Structures and Graph Theory
Time: 2 hour

| Q1. | Choose the correct option for following questions. All the Questions are compulsory and carry equal marks. |
| :---: | :---: |
| 1. | In a class of 50 students, 20 students play cricket and 16 students play football. It is found that 10 students play both the games. Find out the number of students who play neither of the games. |
| Option A: | 42 |
| Option B: | 24 |
| Option C: | 12 |
| Option D: | 14 |
|  |  |
| 2. | Let $\mathrm{A}=\{1,2,3,4,5,6,7,8\}$. Let xRy whenever y is divisible by x , so R is a |
| Option A: | Equivalence Relation |
| Option B: | Partial Order Relation |
| Option C: | Symmetric |
| Option D: | Neither Equivalence Nor Partial Order Relation |
|  |  |
| 3. | $\left(p^{\wedge} p\right)^{\wedge}\left(p \quad\left(q^{\wedge} q\right)\right)$ is equivalent to |
| Option A: | p q |
| Option B: | q p |
| Option C: | $\mathrm{p}^{\wedge} \mathrm{q}$ |
| Option D: | None of the above |
|  |  |
| 4. | If f and g are onto then function (gof) is ? |
| Option A: | one to one |
| Option B: | one to many |
| Option C: | into |
| Option D: | onto |
|  |  |
| 5. | Consider P : Food is good, Q: Service is good, R: Restaurant is 5-star. Write the symbolic notation of the statement " It is not true that 5 star rating always means good food and good service" |
| Option A: | ( $\mathrm{P}^{\wedge} \mathrm{Q}$ )--> R |
| Option B: | $\sim(\mathrm{R} \rightarrow(\mathrm{P} \wedge \mathrm{Q}))$ |
| Option C: | $\mathrm{R} \rightarrow \sim(\mathrm{P} \wedge \mathrm{Q})$ |
| Option D: | $\mathrm{P}^{\wedge} \sim \mathrm{Q}$ |


|  |  |
| :---: | :---: |
| 6. | A is a semigroup (A,*) that has an identity element. |
| Option A: | Cyclic group |
| Option B: | Lattice |
| Option C: | Poset |
| Option D: | Monoid |
|  |  |
| 7. | A graph having all vertices with equal degree is known as |
| Option A: | Regular Graph |
| Option B: | Euler Graph |
| Option C: | Simple Graph |
| Option D: | Hamiltonian Graph |
|  |  |
| 8. | Which of the following is a Tautology? |
| Option A: | $(\sim p \vee p)^{\wedge} \mathrm{q}$ |
| Option B: | $(\mathrm{p} \vee \mathrm{q}) \rightarrow(\mathrm{p} \rightarrow \mathrm{q})$ |
| Option C: | $\left((p \vee q)^{\wedge} \sim p\right) \rightarrow q$ |
| Option D: | $(\sim p \vee \sim q) \rightarrow(p \rightarrow q)$ |
|  |  |
| 9. | A graph has an Euler circuit if |
| Option A: | it is connected and has an even number of vertices. |
| Option B: | it is connected and has an even number of edges. |
| Option C: | it is connected and every vertex has an odd degree. |
| Option D: | every vertex has even degree |
|  |  |
| 10. | Let $f$ and $g$ be the function from the set of integers to itself, defined by $f(x)=3 x+1$ and $g(x)=4 x+4$. Then the composition of $f$ and $g$ is |
| Option A: | $12 \mathrm{x}+4$ |
| Option B: | $12 \mathrm{x}+5$ |
| Option C: | $12 x+13$ |
| Option D: | $12 \mathrm{x}+8$ |
|  |  |
| 11. | k 10 is a complete graph on 10 vertices and will have edges. |
| Option A: | 45 |
| Option B: | 54 |
| Option C: | 40 |
| Option D: | 42 |
|  |  |
| 12. | Solution of linear homogenous recurrence relation: $a_{n}=3 a_{n-1}-2 a_{n-2}$ with $a_{0}=1, a_{1}=3, n \geq 2$ |
| Option A: | $a_{n}=(-1)+2^{n}$ |
| Option B: | $a_{n}=(-1)+3.2^{n}$ |
| Option C: | $a_{n}=(-1)(-1)^{n}+2^{n}$ |
| Option D: | $a_{n}=(-1)+2.2^{n}$ |
|  |  |
| 13. | Let A be a finite set of size n , the number of elements in the power set of A is |


| Option A: | $2^{\text {n }}$ |
| :---: | :---: |
| Option B: | $\mathrm{n}^{2}$ |
| Option C: | $(2 \mathrm{n})^{2}$ |
| Option D: | $2^{2 n}$ |
| 14. | The transitive closure of the relation $R=\{(a, b),(b, c),(c, d)(e, d)\}$ on set $A=\{a, b, c, d, e\}$ is |
| Option A: | $\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{c}),(\mathrm{c}, \mathrm{d}),(\mathrm{e}, \mathrm{d}),(\mathrm{a}, \mathrm{c})$ \} |
| Option B: | $\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{c}),(\mathrm{c}, \mathrm{d}),(\mathrm{e}, \mathrm{d}),(\mathrm{a}, \mathrm{c}),(\mathrm{a}, \mathrm{d}),(\mathrm{b}, \mathrm{d})\}$ |
| Option C: | $\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{c}),(\mathrm{c}, \mathrm{d}),(\mathrm{e}, \mathrm{d}),(\mathrm{a}, \mathrm{c}),(\mathrm{a}, \mathrm{d})$ \} |
| Option D: | $\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{c}),(\mathrm{c}, \mathrm{d}),(\mathrm{d}, \mathrm{e}),(\mathrm{a}, \mathrm{c}),(\mathrm{a}, \mathrm{d})\}$ |
| 15. | What is the correct translation of the following statement into mathematical logic? "Some real numbers are rational" |
| Option A: | $\exists \mathrm{x}(\mathrm{real}(\mathrm{x}) \mathrm{v}$ rational(x)) |
| Option B: | $\exists \mathrm{x}(\operatorname{real}(\mathrm{x}) \wedge \operatorname{rational}(\mathrm{x})$ ) |
| Option C: | $\forall \mathrm{x}(\operatorname{real}(\mathrm{x}) \rightarrow \operatorname{rational}(\mathrm{x})$ ) |
| Option D: | $\exists \mathrm{x}(\operatorname{rational}(\mathrm{x}) \rightarrow \operatorname{real}(\mathrm{x})$ ) |
| 16. | The minimum number of edges in a connected graph with n vertices is |
| Option A: | n-1 |
| Option B: | n |
| Option C: | $\mathrm{n}+1$ |
| Option D: | $\mathrm{n}+2$ |
| 17. | The following graph is |
| Option A: | Bipartite Graph |
| Option B: | Complete Bipartite Graph |
| Option C: | Mixed Graph |
| Option D: | Simple Graph |
| 18. | What is the minimum number of students required in a class to be sure that at least 6 will receive the same grade , if there are five possible grades $A, B, C, D$ and E. |
| Option A: | 62 |
| Option B: | 66 |
| Option C: | 26 |
| Option D: | 22 |
| 19. | Which of the following four subset of integers N is not closed under the operation of multiplication. |
| Option A: | $\mathrm{A}=\{0,1\}$ |
| Option B: | $\mathrm{F}=\{2,4,6, \ldots$. |


| Option C: | $\mathrm{B}=\{1,2\}$ |
| :---: | :--- |
| Option D: | $\mathrm{E}=\{1,3,5, \ldots \ldots\}$ |
|  |  |
| 20. | The <br> corresponding bits. |
| Option A: | Hamming code |
| Option B: | Hamming distance |
| Option C: | Hamming rule words is the number of differences between |
| Option D: | Hamming parity checks |


| $\begin{gathered} \text { Q2. } \\ \text { (20 Marks) } \\ \hline \end{gathered}$ | Solve any Four questions out of Six. 5 marks each |
| :---: | :---: |
| A | Find the CNF form of $(\sim a \rightarrow b)^{\wedge}(a \leftrightarrow b)$ |
| B | Define the following with example <br> 1.Ring 2. Bipartite Graph 3.Chain 4.Semigroup 5. Sublattice |
| C | Define Euler Path and Euler Circuit. Check whether Euler Path , Euler Circuit exist in the following graphs. |
| D | Consider $\mathrm{G}=\{1,2,3,4,5,6\}$ under the multiplication modulo 7. <br> i) Find multiplication table of $G$ <br> ii) Find $2^{-1}, 3^{-1}, 6^{-1}$ <br> iii) Is G cyclic? |
| E | Prove using Mathematical Induction that $n^{3}+2 n$ is divisible by 3 for all $n>=1$ |
| F | Define and give examples of injective surjective and bijective functions. Check the injectivity and surjectivity of the following function $\mathrm{f}: \mathrm{N} \quad \mathrm{N}$ given by $\mathrm{f}(\mathrm{x})=\mathrm{x}^{3}$ |


| Q3. <br> (20 Marks) | Solve any Two Questions out of Three . 10 marks each |
| :---: | :--- |
| A | Let D60 be the poset consisting of all the positive divisors of 60 <br> under the partial order of divisibility. |


|  | (a) Write down the elements of D60? <br> (b) Draw the Hasse Diagram of D60. <br> (c) Define Lattice. Is D60 a lattice? Give a reason for your answer |
| :---: | :--- |
| B | Define Isomorphic Graph. Draw K6 and K3,3 graphs . Find whether they <br> are Isomorphic or not? |
| C | Let A= $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ and let <br> $\mathrm{R}=\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{a}, \mathrm{c}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b}),(\mathrm{c}, \mathrm{a}),(\mathrm{b}, \mathrm{c}),(\mathrm{c}, \mathrm{b}),(\mathrm{c}, \mathrm{c}),(\mathrm{d}, \mathrm{d})\}$. Show that R is a <br> equivalence relation and determine the equivalence classes and find the <br> rank of R. |

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Program: Computer Engineering
Curriculum Scheme: Rev 2019
Examination: SE Semester III (DSE)
Course Code: CSC302 and Course Name: Discrete Structures and Graph Theory

| Question <br> Number | Correct Option (Enter either ' $A$ ' or ' $B$ ' or ' $C^{\prime}$ or ' $D$ ') |
| :---: | :---: |
| Q1. | B |
| Q2. | B |
| Q3. | C |
| Q4 | D |
| Q5 | B |
| Q6 | D |
| Q7 | A |
| Q8. | C |
| Q9. | D |
| Q10. | C |
| Q11. | A |
| Q12. | D |
| Q13. | A |
| Q14. | B |
| Q15. | B |
| Q16. | A |
| Q17. | B |
| Q18. | C |
| Q19. | C |
| Q20. | B |

## Q. 2 A) Define CNF form

1mark
Derivation Steps
Ans: $(\mathrm{a} \vee \mathrm{b})^{\wedge}(\sim \mathrm{a} \vee \mathrm{b})^{\wedge}(\sim \mathrm{b} \vee \mathrm{a})$
B)

For each correct definition 1 mark 5marks

## C)

Define a Euler Path 1 mark

Define Euler Circuit. 1 mark

G1 has two vertices of odd degree and and the rest of them have even degree. So this graph has an Euler path but not an Euler circuit. The path starts and ends at the vertices of odd degree. The path is- a,c,d,a,b,d.

G2 has four vertices all of even degree, so it has a Euler circuit. The circuit is $-\mathrm{a}, \mathrm{d}, \mathrm{b}, \mathrm{a}, \mathrm{c}, \mathrm{d}, \mathrm{a}$.

$$
3 \text { marks }
$$

D) Multiplication table of G

2 marks

| $\mathrm{x}_{7}$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 4 | 6 | 1 | 3 | 5 |
| 3 | 3 | 6 | 2 | 5 | 1 | 4 |
| 4 | 4 | 1 | 5 | 2 | 6 | 3 |
| 5 | 5 | 3 | 1 | 6 | 4 | 2 |
| 6 | 6 | 5 | 4 | 3 | 2 | 1 |

inverse of $2^{-1}$ is $4,3^{-1}$ is $5,6^{-1} \quad$ is $6 \quad 2$ mark
$G$ is cyclic
1 mark
$\begin{array}{ll}\text { Define MI } & 1 \text { mark } \\ \text { Correct proof } & 4 \text { marks }\end{array}$
F) $\begin{array}{ll}\text { Definition with example } & 3 \text { marks } \\ \text { example is injective not surjective } & 2 \text { marks }\end{array}$

## Q. 3 A)

a) Elements of D60
b) correct Haase diagram
c) Lattice Definition

Reason for lattice

2 marks
3 mark each
2 mark
3 marks
B ) Define Isomorphic graph ..... 2 marks
Draw K6 . 3 marks
Draw K3,3
3 marks
(graphs are not isomorphic)
2 mark

## C)

Definition of Equivalence relation 2 marks
Show that R is Equivalence
3 marks
Find equivalence classes
3 marks
$[\mathrm{a}]=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\} \quad[\mathrm{b}]=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\} \quad[\mathrm{c}]=\{\mathrm{c}, \mathrm{a}, \mathrm{b}\} \quad[\mathrm{d}]=\{\mathrm{d}\}$
$\begin{array}{cc}\text { Find rank of } R-\text { Rank definition } & 1 \text { mark } \\ \text { Rank of } R \text { is } 2 & 1 \text { mark }\end{array}$

