

K. J. Somaiya Institute of Engineering and Information Technology, Sion, Mumbai-22
(Autonomous College Affiliated to University of Mumbai)

End Semester Exam

Nov - Dec 2021

Program: B.Tech EXTC Engineering

Examination: SY Semester: III

Course Code: IUEXC301

Course Name: Applications of Mathematics in Engineering-I

Duration: 03 Hours

Max. Marks: 60

Instructions:

- (1) All questions are compulsory.
- (2) Draw neat diagrams wherever applicable.
- (3) Assume suitable data, if necessary.

Q. No.	Question	Max. Marks	CO	BT Level
Q 1	Solve any six questions out of eight:	12		
i)	Find the Laplace transform of $(e^{2t} + 4t^3 - \sin 2t \cos 3t)$.	2	CO1	3
ii)	Find the inverse Laplace transform of $\frac{s+2}{s^2-4s+13}$	2	CO2	3
iii)	Obtain the inverse Laplace transform of $\left(\frac{1}{(s-3)(s-1)}\right)$	2	CO2	3
iv)	Find the derivative of the function $f(z) = x^2 - y^2 - i 2xy$	2	CO4	4
v)	Find p such that the function $f(z) = r^2 \cos 2\theta + i r^2 \sin p\theta$ is analytic.	2	CO4	3
vi)	Find the Fourier cosine transform of $f(x) = e^{-ax}$, $a > 0$, $x > 0$	2	CO3	3
vii)	If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ find the eigenvalues of the matrix $A^3 + 5A + 8I$	2	CO5	3
viii)	Determine the constant a so that the vector $\vec{F} = (x + 3y^3)\hat{i} + (2y + 2z^2)\hat{j} + (x^2 + az)\hat{k}$ is solenoidal.	2	CO6	3

Q.2	Solve any four questions out of six.	16		
i)	Given $f(t) = \begin{cases} t+1, & 0 \leq t \leq 2 \\ 3, & t > 2 \end{cases}$ Find $L(f(t))$	4	CO1	3
ii)	Using the convolution theorem, Find $L^{-1}\left(\frac{4}{(s^2+2s+5)^2}\right)$.	4	CO2	3
iii)	Find the orthogonal trajectories of the family of the curves $e^x \cos y = a$, where a is real constant in the xy plane	4	CO4	3
iv)	Find the Fourier series to represent $f(x) = x^2$ from $-2 < x < 2$.	4	CO3	3
v)	Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point $A(1, -1, -1)$ in the direction of the line AB where B has coordinates $(3, 2, 1)$.	4	CO6	4
vi)	If $A = \begin{bmatrix} \frac{\pi}{2} & \pi \\ 0 & \frac{3\pi}{2} \end{bmatrix}$, find $\sin A$	4	CO5	4
Q.3	Solve any two questions out of three.	16		
i)	a). Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$. b). Find the Laplace transform of $\cos 3t \cos 2t \cos t$.	4 4	CO1	4
ii)	Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ Hence evaluate A^{-1} and A^4	8	CO5	3
iii)	If $\vec{F} = (2xy + z^3)\hat{i} + (x^2)\hat{j} + (3xz^2)\hat{k}$ is conservative then find its scalar potential Φ and also find the work done in moving a particle under this force field from $(1, -2, 1)$ to $(3, 1, 4)$	8	CO6	4

Q.4	Solve any two questions out of three.	16		
i)	Obtain the inverse Laplace transform of the following. a). $\log \left(\frac{s^2+a^2}{s^2+a^2} \right)$. b). $\left(\frac{54}{s^3 (s-3)} \right)$.	4 4	CO2	4
ii)	a). Determine whether the following function is analytic and if so find its derivative $f(z) = (x^3 - 3xy^2 + 2xy) + i(3x^2y - x^2 + y^2 - y^3)$ b). Prove that the following function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find the conjugate harmonic function v .	4 4	CO4	4
iii)	Find the Fourier series to represent the function $f(x) = x - x^2$ for $-\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$	8	CO3	4