

K. J. Somaiya Institute of Engineering and Information Technology, Sion, Mumbai-22
 (Autonomous College Affiliated to University of Mumbai)
End Semester Exam

Nov-DEC 2022
 B.Tech. Program: COMP/IT/AI-DS

Examination: SY Semester: IV
 Course Code: 1UCEC401/1UITC401/1UAIC401

Course Name: Application of Mathematics in Engineering -II

Duration: 03 Hours

Max. Marks: 60

Instructions:

- (1) All questions are compulsory.
- (2) Draw neat diagrams wherever applicable.
- (3) Assume suitable data, if necessary.

		Max. Marks	C O	BT level
Q 1	Solve any six questions out of eight:	12		
i)	Evaluate the integral $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x$.		2	3
ii)	1) State Cayley Hamilton Theorem. 2) State whether the following statement is true or false and justify: If Eigen values of a matrix are distinct then that matrix is not diagonalizable.		1	3
iii)	Define Basis and Dimension of a vector space.		3	3
iv)	If $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$, then find the eigen values of the matrix $A^2 + 6A^{-1} + 2I$.		1	3
v)	Find stationary point of $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$		6	3
vi)	Define simple closed curve and evaluate $\int_C \frac{1}{z+3} dz$ where C is the circle $ z = 1$.		2	3
vii)	Write the standard form of LPP Minimize $Z = -2x_1 + x_2$ Subject to $4x_1 + 5x_2 \geq -4, -3x_1 + 5x_2 \leq 7, x_1, x_2 \geq 0$		5	3
viii)	If for a Poisson variate $m = 2$, find the probabilities of $x = 1$ using recurrence relation of Poisson distribution.		4	3
Q.2	Solve any four questions out of six.	16		
i)	Evaluate $\int_C \frac{7z-1}{(z-3)(z+5)} dz$, where C is the circle $ z = 1$.		2	3
ii)	Using Cayley Hamilton theorem find the inverse of matrix $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.		1	3
iii)	A soap manufacturing company was distributing a particular brand of soap		4	3

	through a large number of retail shops. Before a heavy advertisement campaign, the mean sales per week per shop were 140 dozens. After the campaign a sample of 26 shops was taken and the mean sale was found to be 147 dozens with standard deviation of 16. Can you consider the advertisement effective?			
iv)	Obtain the dual of the following L.P.P. $\text{Minimise } z = 2x_1 - x_2 + 3x_3$ Subject to $x_1 - 2x_2 + x_3 \geq 4$ $2x_1 + x_3 \leq 10$ $x_1 + x_2 + 3x_3 = 20$ $x_1, x_3 \geq 0, x_2$ is unrestricted.		5	3
v)	Check whether $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, 0\right)$ are orthogonal with respect to the Euclidean inner product.		3	3
vi)	For the following function find the stationary point and Hessian Matrix at that stationary point $z = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 100$		6	3
Q.3	Solve any two questions out of three.	16		
i)	Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into orthonormal bases where $u_1 = (1, 1, 1), u_2 = (-1, 1, 0), u_3 = (1, 2, 1)$.		3	8
ii)	Obtain Taylor's and Laurent's series expansions of $f(z) = \frac{z-1}{z^2-2z-3}$. Also indicate the regions of convergence.		2	8
iii)	Use Lagrange's multipliers method to solve the following NLPP. Optimise $z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$ Subject to $x_1 + x_2 + x_3 = 10, x_1, x_2, x_3 \geq 0$		6	8
Q.4	Solve any two questions out of three.	16		
i)	Check whether matrix A given below is diagonalizable. If it is diagonalizable, find the diagonal matrix and the transforming matrix. $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$		1	8
ii)	Solve the following LPP using Big-M method Maximize $z = 3x_1 - x_2$ Subject to $2x_1 + x_2 \leq 2, x_1 + 3x_2 \geq 3, x_2 \leq 4, x_1, x_2 \geq 0$		5	8
iii)	(A) The marks obtained by 1000 students in an examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be (i) between 60 and 75 (ii) more than 75. (B) If X is a Poisson variate and $P(X = 0) = 6P(X = 3)$, find $P(X = 2)$.		4	8