K. J. Somaiya Institute of Technology, Sion, Mumbai-22
(Autonomous College Affiliated to University of Mumbai)

Nov – Dec 2023
Program: B.Tech. Scheme: III
Examination: FY Semester: I
Course Code: BSC101 and Course Name: Engineering Mathematics I

Date of Exam: 22/12/2023 Duration: 2.5 Hours Max. Marks: 60

Instructions:
(1) All questions are compulsory.
(2) Assume suitable data, if necessary.

		Max. Marks	CO	BT level
Q 1	Solve any six questions out of eight:	12		
i)	Using De'Moivre's theorem prove that $(1+i\sqrt{3})^8 + (1-i\sqrt{3})^8 = -2^8$.	2	1	A
ii)	If $tanhx = \frac{1}{2}$, find the value of x.	2	2	A
iii)	If $z = x^y + y^x$, find $\frac{\partial^2 z}{\partial x \partial y}$.	2	3	A
iv)	Find y_n, if $y = e^{2x} sin^2 x$.	2	4	A
v)	If A is Hermitian Matrix, then prove that iA is Skew – Hermitian Matrix.	2	5	A
vi)	Determine the interval (a, b), in which solution to equation $3x^3 - 4x^2 - 4x - 7 = 0$ lies.	2	6	A
vii)	If $u = \frac{x^2y^2}{\sqrt{x}+\sqrt{y}}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3	A
viii)	Find the rank by reducing the following matrix to echelon form. $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$	2	5	A
Q.2	Solve any four questions out of six.	16		
i)	If $\sin 6x = a\cos^5 x \sin x + bcos^3 x \sin^3 x + c\cos x \sin^5 x$, find the values of a, b, and c.	4	1	A
ii)	Prove that $log(e^{i\alpha} + e^{i\beta}) = log\left\{2\cos\left(\frac{\alpha-\beta}{2}\right)\right\} + i\left\{\frac{\alpha+\beta}{2}\right\}$	4	2	A

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iii)	Find a, b, c if the matrix $\frac{1}{3}\begin{bmatrix} a & b & c \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ is orthogonal.	4	3	A
iv)	If $z = \tan^{-1}\left(\frac{x}{y}\right)$, $x = 2t, y = 1 - t^2$ prove that $\frac{dz}{dt} = \frac{2}{1+t^2}$.	4	4	A
v)	If $y = \frac{x^2}{(x-1)^2(x+2)}$, find y_n.	4	5	A
vi)	Using Newton Raphson method, find a root of $x^3 - 2x - 5 = 0$ correct to three decimal places, start with $x_0 = 3$.	4	6	A
Q.3	Solve any two questions out of three.	16		
i)	If $\alpha, \alpha^2, \alpha^3, \alpha^4$ are the roots of $x^5 - 1 = 0$ then find α and prove that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$.	8	1	A
ii)	Investigate for what value of λ, the equations below has a solution and solve them in eachcase. $x + 2y + z = 3$, $x + y + z = \lambda$, $3x + y + 3z = \lambda^2$	8	3	A
iii)	Find the stationary points and extreme values of $x^3 + y^3 - 63(x + y) + 12xy$.	8	5	A
Q.4	Solve any two questions out of three.	16		
i)	If $\tan(\alpha + i\beta) = x + iy$ then prove that (i) $x^2 + y^2 + 2x\cot 2\alpha = 1$ (ii) $x^2 + y^2 - 2x\coth 2\beta + 1 = 0$	8	2	A
ii)	If $u = x^3 \sin^{-1}\left(\frac{y}{x}\right) + x^4 \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at (1,1) .	8	4	A
iii)	Using Gauss – Seidel method solve the following equations up to 4 – iterations. $15x + 2y + z = 22$; $x + 14y + 2z = 35$; $x + 2y + 15z = 50$	8	6	A

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