K. J. Somaiya Institute of Technology, Sion, Mumbai-22
(Autonomous College Affiliated to University of Mumbai)

April – May 2022-23
Program: B.Tech. Scheme: II
Examination: FY Semester: I
Course Code: BSC101 and Course Name: Engineering Mathematics I

Date of Exam: 22/11/2022 Duration: 2.5 Hours Max. Marks: 60

Instructions:
(1) All questions are compulsory.
(2) Draw neat diagrams wherever applicable.
(3) Assume suitable data, if necessary. All logarithms are taken with base e.

		Max. Marks	CO	BT level
Q 1	**Solve any six questions out of eight:**	12		
i)	Prove that $(1+i\sqrt{3})^8 + (1-i\sqrt{3})^8 = -2^8$.	02	C01	A
ii)	If $x+\frac{1}{x} = 2\cos\theta$, $y+\frac{1}{y} = 2\cos\phi$, then find the value of $xy+\frac{1}{xy}$.	02	C01	A
iii)	Show that $\left(\frac{1+\tanh x}{1-\tanh x}\right)^3 = \cosh 6x + \sinh 6x$.	02	C02	A
iv)	Separate into real and imaginary parts of $i^{\log(1+i)}$.	02	C02	A
v)	If $u = e^{xy}$, where $x = tcost, y = tsint$ find $\frac{du}{dt}$ at $t = \frac{\pi}{2}$.	02	C03	A
vi)	If $u = \left(\frac{x}{y}\right)^{\frac{y}{x}}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	02	C03	A
vii)	Find all the stationary points of $f(x,y) = xy(3-x-y)$.	02	C04	A
viii)	Determine whether the following matrix is unitary or not $A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$.	02	C05	A
Q.2	**Solve any four questions out of six.**	16		
i)	If $\cos 4\theta = a\cos^4\theta + b\cos^2\theta\ \sin^2\theta + c\sin^4\theta$, find the value of a, b and c.	04	C01	A
ii)	Prove that $\sinh^{-1}(\tan\theta) = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.	04	C02	A

K. J. Somaiya Institute of Technology, Sion, Mumbai-22
(Autonomous College Affiliated to University of Mumbai)

April – May 2022-23
Program: B.Tech. Scheme: II
Examination: FY Semester: I
Course Code: BSC101 and Course Name: Engineering Mathematics I

Date of Exam: 22/04/2023 Duration: 2.5 Hours Max. Marks: 60

iii)	If $u = \sinh^{-1}\left(\frac{x^3+y^3}{x^2+y^2}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\tanh^3 u$.	04	C03	A
iv)	If $y = 2x\sqrt{1-x^2}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-4)y_n = 0$.	04	C04	A
v)	Find the points on the curve $xy^2 = 54$ which are nearest to the origin.	04	C04	A
vi)	Find the rank of the following matrix by reducing to normal form. $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$.	04	C05	A
Q.3	**Solve any two questions out of three.**	16		
i)	If $\cos\alpha + \cos\beta + \cos\gamma = 0$, and $\sin\alpha + \sin\beta + \sin\gamma = 0$ prove that (i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$	08	CO1	A
ii)	Show that $\tan^{-1}\left(\frac{x+iy}{x-iy}\right) = \frac{\pi}{4} + \frac{i}{2}\log\left(\frac{x+y}{x-y}\right)$.	08	CO2	A
iii)	Investigate for what value of λ, the equations below has a solution and solve them in each case. $x + 2y + z = 3,\ x + y + z = \lambda,\ 3x + y + 3z = \lambda^2$.	08	CO5	A
Q.4	**Solve any two questions out of three.**	16		
i)	If $u = \frac{x^4+y^4}{x^2y^2} + x^6\tan^{-1}\left(\frac{x^2+y^2}{x^2+2xy}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at $(x,y) = (1,2)$.	08	CO3	A
ii)	The temperature $T(x,y,z)$ at any point in the space is $T = 400xyz^2$. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = 1$.	08	CO4	A
iii)	Find for what values of k, the set of equations $2x - 3y + 6z - 5t = 3, y - 4z + t = 1, 4x - 5y + 8z - 9t = k$, has 1) no solution, 2) infinitely many solutions, Also find solution in each case.	08	CO5	A