Supplimentary Exam

K. J. Somaiya Institute of Technology, Sion, Mumbai-22
(Autonomous College Affiliated to University of Mumbai)

Feb-March-2024

Program: B.Tech. Scheme: III
Examination: FY Semester: I
Course Code: BSC101 and Course Name: Engineering Mathematics I

Date of Exam: 27/02/2024 Duration: 2.5 Hours Max. Marks: 60

Instructions:
(1) All questions are compulsory.
(2) Assume suitable data, if necessary.

		Max. Marks	CO	BT level
Q 1	Solve any six questions out of eight:	12		
i)	If $x = \cos\theta + i\sin\theta$, then find the value of $x^n + \frac{1}{x^n}$.	2	1	A
ii)	Find the value of $\tanh x$, if $x = \log\sqrt{2}$.	2	2	A
iii)	If $u = \frac{1}{r}$ and $r = \sqrt{x^2 + y^2 + z^2}$, find the value of $\frac{\partial^2 u}{\partial x^2}$.	2	3	A
iv)	Find y_n at $x = 0$, if $y = e^{2x}x^2$.	2	4	A
v)	Determine whether following matrix A, is Unitary? $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$	2	5	A
vi)	Determine the interval (a, b), in which solution to equation $2x - 3\sin x - 5 = 0$ lies.	2	6	A
vii)	If $u = \sqrt{x} + \sqrt{y}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3	A
viii)	Reduce the following matrix to echelon form. $\begin{bmatrix} 1 & 2 & 1 & 3 \\ 3 & 4 & -3 & -1 \\ -1 & 0 & 5 & 7 \end{bmatrix}$	2	5	A
Q.2	Solve any four questions out of six.	16		
i)	Show that the continued product of all the values of $i^{\frac{2}{3}}$ is -1.	4	1	A
ii)	Prove that $\cosh^{-1}\sqrt{(1+x^2)} = \sinh^{-1}x$.	4	2	A
iii)	If $u = lx + my$, $v = mx - ly$, prove that $\left(\frac{\partial u}{\partial x}\right)_y \times \left(\frac{\partial x}{\partial u}\right)_v = \frac{l^2}{l^2+m^2}$.	4	3	A
iv)	Find all the stationary points (& not extreme values) of the function $xy(3 - x - y)$.	4	4	A

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v)	Find a, b, c if following matrices are orthogonal. $\frac{1}{3}\begin{bmatrix} a & b & c \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$	4	5	A
vi)	Solve the following equations by Gauss-Seidel method $10x + y + z = 12, 2x + 10y + z = 13, 2x + 2y + 10z = 14.$	4	6	A
Q.3	Solve any two questions out of three.	16		
i)	Prove using D' Moivres Theorem $\cos^5 x \sin^3 x = -\frac{1}{2^7}(\sin 8x + 2\sin 6x - 2\sin 4x - 6\sin 2s)$	8	1	A
ii)	If $u = x^3 \sin^{-1}\left(\frac{y}{x}\right) + x^4 \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at (1,1) .	8	3	A
iii)	Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 1$.	8	5	A
Q.4	Solve any two questions out of three.	16		
i)	If $\tan(\alpha + i\beta) = x + iy$ then prove that a. $x^2 + y^2 + 2x\cot 2\alpha = 1$ b. $x^2 + y^2 - 2x\coth 2\beta = -1.$	8	2	A
ii)	Investigate for what value of λ, the equations below has a solution and solve them in eachcase. $x + 2y + z = 3,\ x + y + z = \lambda,\ 3x + y + 3z = \lambda^2$	8	4	A
iii)	Using Newton-Raphson method find the approximate value of the root of the equation $2x^3 - 3x + 4 = 0$ lying between -2 and -1 and correct upto four decimal places.	8	6	A

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