

K. J. Somaiya Institute of Technology, Sion, Mumbai-22
(Autonomous College Affiliated to University of Mumbai)

Supplementary Exam

Jan - Feb 2024
 Program: B.Tech. Scheme: II
 Examination: FY Semester: I
 Course Code: BSC101 and Course Name: Engineering Mathematics I

Date of Exam: 27/1/24 Duration: 2.5 Hours Max. Marks: 60

Instructions:

- (1) All questions are compulsory.
 (2) Assume suitable data, if necessary.

		Max. Marks	CO	BT level
Q 1	Solve any six questions out of eight:	12		
i)	Write the polar form of $z = 1 + i\sqrt{3}$	2	1	A
ii)	If $x + \frac{1}{x} = 2 \cos \theta$, $y + \frac{1}{y} = 2 \cos \phi$, then find the value of $xy - \frac{1}{xy}$.	2	1	A
iii)	If $\sin(\alpha - i\beta) = x + iy$ then prove that $\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$	2	2	A
iv)	Solve the equation $17 \cosh x + 18 \sinh x = 1$ for real values of x	2	2	A
v)	If $u = x^3 - y^3$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.	2	3	A
vi)	Find y_n at $x = 0$, if $y = 2^x x^2$.	2	4	A
vii)	Determine whether matrix $A = \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ is Orthogonal?	2	5	A
viii)	Reduce the matrix $A = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 1 & -1 & 0 & 3 \\ 3 & 5 & 1 & 6 \end{bmatrix}$ to row-echelon form.	2	5	A
Q.2	Solve any four questions out of six.	16		
i)	Find the continued product of all the values of $\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^{3/4}$	4	1	A
ii)	If $\frac{(1+i)^{x+iy}}{(1-i)^{x-iy}} = \alpha + i\beta$, find α and β	4	2	A
iii)	If $z(x+y) = x^2 + y^2$, prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$	4	3	A

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iv)	If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3} + \log\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$.	4	3	A
v)	If $u = y^2 - 4ax, \dot{x} = at^2, y = 2at$, show that $\frac{du}{dt} = 0$	4	4	A
vi)	Find the values of P for which the following matrix A will have (i) rank 1, (ii) rank 2, (iii) rank 3, where A is $\begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$	4	5	A
Q.3	Solve any two questions out of three.	16		
i)	Use De Moivre's Theorem to show that $\tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$ and hence deduce that $5 \tan^4 \frac{\pi}{10} - 10 \tan^2 \frac{\pi}{10} + 1 = 0$	8	1	A
ii)	If $\tan z = \frac{i}{2}(1 - i)$, prove that $z = \frac{1}{2} \tan^{-1} 2 + \frac{i}{4} \log(5)$	8	2	A
iii)	If $y = \sin(m \sin^{-1} x)$, prove that, $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.	8	4	A
Q.4	Solve any two questions out of three.	16		
i)	If $u = x^3 \sin^{-1} \frac{y}{x} + x^4 \tan^{-1} \frac{y}{x}$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ at $x = 1, y = 1$	8	3	A
ii)	Find the maxima and minima of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$	8	4	A
iii)	Investigate for what values of a, b the following linear equations $x + 2y + 3z = 4, x + 3y + 4z = 5, x + 3y + az = b$, have (i) no solution, (ii) a unique solution, (iii) An infinite number of solutions.	8	5	A