

## University of Mumbai

### Examination 2020 under cluster 4 (Lead College: PCE, New Panel)

Examinations Commencing from 23<sup>rd</sup> December 2020 to 6<sup>th</sup> January 2021 and from 7<sup>th</sup> January 2021  
to 20<sup>th</sup> January 2021

Program: Computer Engineering

Curriculum Scheme: Rev2019

Examination: SE Semester III

Course Code: CSC302 and Course Name: Discrete Structures and Graph Theory

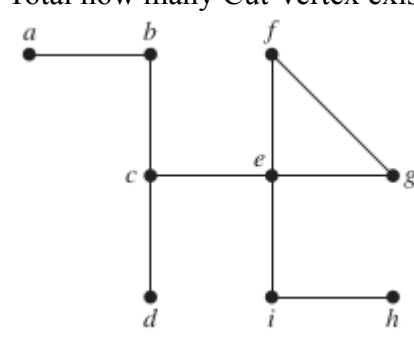
Time: 2 hour

Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks
1.	Let $A = \{2,3,4,5,6\}$ and let $R_1, R_2$ be relations on $A$ such that $R_1 = \{(a,b) \mid a-b=2\}$ and $R_2 = \{(a,b) \mid a+1=b \text{ or } a=2b\}$ Find the composite relation $R_2.R_1$ ?
Option A:	$\{(4,3),(5,4),(6,2),(6,5)\}$
Option B:	$\{(3,2),(5,4),(4,3)\}$
Option C:	$\{(5,2),(6,3)\}$
Option D:	$\{(2,3),(3,4),(4,5),(5,6)\}$
2.	Which of the following is the correct representation of the sentence "Someone is liked by everyone"?
Option A:	$(\exists x)(\exists y) \text{ likes}(x,y)$
Option B:	$(\forall x)(\forall y) \text{ likes}(x,y)$
Option C:	$(\exists y)(\forall x) \text{ likes}(x,y)$
Option D:	$(\forall x)(\exists y) \text{ likes}(x,y)$
3.	Draw the Hasse diagram of $D_{30}$ . i) It is Complemented Lattice ii) It is Distributive Lattice Which of the above statement is True?
Option A:	Only i
Option B:	Only ii
Option C:	Both i and ii
Option D:	Neither i nor ii
4.	Consider the set $N$ of positive integers, and let $*$ denote the operation of least common multiple(lcm) on $N$ . Which of the following sentence is True?
Option A:	$(N,*)$ is not a Semi group.
Option B:	$(N,*)$ is commutative Semi group
Option C:	$(N,*)$ is not commutative Semi group.
Option D:	None of the Above.

5.	How many two digits or three digits numbers can be formed using the digits 1,2,3,4,5,6,7,8 and 9 , if no digits are repeated ?
Option A:	210
Option B:	24
Option C:	212
Option D:	252
6.	Consider the following subsets of the positive integers N. Which of the following is not closed under multiplication operation?
Option A:	$A=\{0,1\}$
Option B:	$E=\{1,3,5,\dots\}$
Option C:	$C=\{x: x \text{ is prime}\}$
Option D:	$F=\{0,1,2\}$
7.	If every vertex of simple graph has same degree it is called as _____.
Option A:	Bipartite Graph
Option B:	Regular Graph
Option C:	Planner Graph
Option D:	Sub graph
8.	The less than relation, $<$ , on real is
Option A:	A Partial ordering since it is asymmetric and reflexive.
Option B:	A partial ordering since it is anti-symmetric and reflexive.
Option C:	Not a partial ordering because it is not asymmetric and not reflexive.
Option D:	Not a partial ordering because it is not anti-symmetric and not reflexive.
9.	Consider set of integers from 1 to 250. Find how many of these numbers are divisible by 5 or 6 but not by 8?
Option A:	83
Option B:	69
Option C:	100
Option D:	31
10.	Consider $G=\{1,5,7,11,17\}$ under multiplication modulo 18. Find inverse of 5, 7 and 17 ?
Option A:	11,17 and 13
Option B:	11,13 and 17
Option C:	11 , 17 and 7
Option D:	13,11 and 7
11.	The following graph is _____.
Option A:	Bipartite Graph
Option B:	Complete Bipartite Graph

Option C:	Eulerian Graph
Option D:	Eulerian but not Bipartite Graph
12.	The set of integers $Z$ with binary operation ‘*’ defined as $a*b=a+b+1$ for $a,b \in Z$ , is a group. The identity element of this group is
Option A:	0
Option B:	1
Option C:	-1
Option D:	12
13.	How many persons must be chosen in order that at least five of them will have birthdays in the same calendar month?
Option A:	28
Option B:	69
Option C:	49
Option D:	52
14.	<p>Which of the following is true for above graph?</p> <p>i) It is Eulerian Graph</p> <p>ii) It is Hamiltonian Graph</p>
Option A:	Only i
Option B:	Only ii
Option C:	Both i and ii
Option D:	Neither i nor ii
15.	A Poset in which every pair of elements has both a least upper bound and a greatest lower bound is termed as
Option A:	Walk
Option B:	Trail
Option C:	Sub lattice
Option D:	Lattice
16.	State the type of function for following example “To each country assign the number of people living in the country”
Option A:	Many-One
Option B:	One-Many
Option C:	One-One
Option D:	Many-Many

17.	Let P: We should be trustworthy. Q: We should be committed. R: We should be overconfident. Then 'We should be trustworthy or committed but not overconfident.' is best represented by?
Option A:	$P \vee Q \wedge R$
Option B:	$\sim P \vee \sim Q \vee R$
Option C:	$P \vee Q \wedge \sim R$
Option D:	$P \wedge \sim Q \wedge R$
18.	Total how many Cut Vertex exists in the following graph? 
Option A:	2
Option B:	4
Option C:	3
Option D:	1
19.	The binary relation $\{(a,a), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b)\}$ on the set $\{a,b,c\}$ is
Option A:	irreflexive, symmetric and transitive
Option B:	reflexive, symmetric and transitive
Option C:	irreflexive and antisymmetric
Option D:	neither reflexive, nor irreflexive but transitive
20.	Which rule of inference is used in this argument? "No humans can fly. John is human. Therefore John can not fly."
Option A:	Universal instantiation
Option B:	Existential instantiation
Option C:	Universal generalization
Option D:	Existential generalization

<b>Q2</b>	
A	<b>Solve any Two</b> <span style="float: right;"><b>5 marks each</b></span>
i.	Let $A = \{1, 2, 3, 4, 5\}$ , $R = \{(a, b) \mid (a+b) \text{ is even}\}$ . R is a relation on set A. Check whether R is an equivalence relation?
ii.	$X = \{2, 3, 6, 1, 24, 36\}$ $R \text{ on } X = \{(x, y) \in R, x \text{ divides } y\}$ <ol style="list-style-type: none"> <li>Construct Hasse diagram</li> <li>Maximum and Minimal elements?</li> <li>Give Chain and Ant chains.</li> </ol>

	<p>d) Maximum length of chain?</p> <p>e) Is a poset lattice?</p>
iii.	<p>Define the following with suitable example</p> <p>a) Ring b) Cyclic Group c) Monoid d) Normal Subgroup e) Planner Graph</p>
B	<p><b>Solve any One</b> <span style="float: right;"><b>10 marks each</b></span></p>
i.	<p>Define with example Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit. Determine if following diagram has Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit and state the path/circuit.</p>
ii.	<p>Find the number of code word generated by the parity check matrix H given below. Find all the code words generated.</p> $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

<b>Q3.</b>	
A	<p><b>Solve any Two</b> <span style="float: right;"><b>5 marks each</b></span></p>
i.	<p>Define Isomorphic Graph. Determine if following graphs G1 and G2 are isomorphic or not.</p>
ii.	<p>Convert into CNF: <math>((P \square Q) \square R)</math></p>
iii.	<p>Functions f,g,h are defined on a set <math>X = \{a,b,c\}</math> as</p> <p><math>f = \{(a,b), (b,c), (c,a)\}</math></p> <p><math>g = \{(a,b), (b,a), (b,b)\}</math></p> <p><math>h = \{(a,a), (b,b), (c,a)\}</math></p> <p>i) Find fog, gof. Are they equal?</p> <p>ii) Find fogoh and fohog?</p>

B	<b>Solve any One</b>	<b>10 marks each</b>
i.	Prove that $(\mathbb{Z}_5, +)$ is a Abelian group.	
ii.	Solve the recurrence relation for Fibonacci sequence 1,1,2,3,5,8,13.	

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<b>Question Number</b>	<b>Correct Option (Enter either 'A' or 'B' or 'C' or 'D')</b>
Q1.	B
Q2.	C
Q3.	C
Q4.	B
Q5.	D
Q6.	C
Q7.	B
Q8.	D
Q9.	B
Q10.	B
Q11.	A
Q12.	C
Q13.	C
Q14.	A
Q15.	D
Q16.	C
Q17.	C
Q18.	B
Q19.	D
Q20.	A

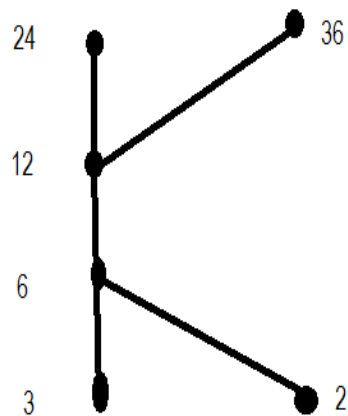
$C = \{(000000), (001011), (010101), (011110), (100111), (101100), (110010), (111001)\}$

Q2. A

- i) Definition of Equivalence relation – 1 marks  
 Prove that relation is equivalence relation – 4 marks

ii)

a. Construct Hasse diagram



- b. Maximum elements = {24,36} and Minimal elements = {3,2}
- c. Chain = {3,6,12,24}, {3,6,12,36}, {2,6,12,24} and {2,6,12,36} and Ant chains = {2,3} and {24,36}
- d. Maximum length of chain? 4
- e. Is a poset lattice? No  
 As (2 and 3) has no lower bound and (24,36) has no upper bound

iii) Definition ½ marks and example ½ mark each

B) Solve any one

i) definition 1/2 mark example 1 mark  
 $11/2 * 4 = 6$  marks for correct path/  
 circuit finding 4 marks.

ii) Ans :



Q3. I) Defintion with example 2 marks

These are not isomorphic graphs ,  
steps 3 marks.

ii) Convert to CNF , apply logic rules,  
and get equivalent form

Ans:  $(P \vee R) \wedge (\sim Q \vee R)$

iii)  $Gof = \{(1,3), (3,1), (2,2)\}$   
 $, gof = \{(1,1), (2,3), (3,2)\}$   
 Fog not equal to gof  
 $Fogoh = \{(1,3), (2,2), (3,3)\}$   
 $Fohog = \{(1,3), (2,2), (3,2)\}$

B solve any one

- i) To prove  $(Z_5, +_5)$  is Abelian group  
Definition of Abelian Group – 2  
marks, stepwise explanation – 8  
marks
- ii) Recurrence relation Fibonacci  
sequence

Sol: Gr: Fibonacci sequence:  $F_n = F_{n-1} + F_{n-2}$  is a homogeneous equation of order 2  
 characteristic Eqn:  $y^2 - y - 1 = 0$   $a=1, b=-1$   

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}$$

$$y_1 = \frac{1+\sqrt{5}}{2}, y_2 = \frac{1-\sqrt{5}}{2} \rightarrow \text{Real \& distinct}$$
 General Solution: Real & distinct:  $F(n) = Ay_1^n + By_2^n$   

$$F(n) = A \left[ \frac{1+\sqrt{5}}{2} \right]^n + B \left[ \frac{1-\sqrt{5}}{2} \right]^n$$
 To Find Constant:  $F_1=1, F_2=1$   
 $n=1, F(1) = A \left[ \frac{1+\sqrt{5}}{2} \right]^1 + B \left[ \frac{1-\sqrt{5}}{2} \right]^1$   

$$1 = \frac{1}{2} [A+B] + \frac{\sqrt{5}}{2} [A-B] \Rightarrow \textcircled{1}$$
 $n=2, F(2) = A \left[ \frac{1+\sqrt{5}}{2} \right]^2 + B \left[ \frac{1-\sqrt{5}}{2} \right]^2$   

$$= A \left[ \frac{1+2\sqrt{5}+5}{4} \right] + B \left[ \frac{1-2\sqrt{5}+5}{4} \right]$$
  

$$= A \left[ \frac{3+\sqrt{5}}{2} \right] + B \left[ \frac{3-\sqrt{5}}{2} \right]$$
  

$$1 = \frac{3}{2} [A+B] + \frac{\sqrt{5}}{2} [A-B] \Rightarrow \textcircled{2}$$

$$\textcircled{2} - \textcircled{1} \Rightarrow 0 = 4B \Rightarrow B = -A$$
 Sub B in  $\textcircled{1}$ ;  $1 = \frac{1}{2} [A-A] + \frac{\sqrt{5}}{2} [A-(-A)] \Rightarrow A = \frac{2}{\sqrt{5}}$   

$$\therefore B = -\frac{2}{\sqrt{5}}$$
 Solution:  $F(n) = \frac{1}{\sqrt{5}} \left[ \frac{1+\sqrt{5}}{2} \right]^n - \frac{1}{\sqrt{5}} \left[ \frac{1-\sqrt{5}}{2} \right]^n$